XXXI. *On the Theory of the Motion of Glaciers.*

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ALMOST all the numerous discussions which have taken place during the last twenty years respecting our theories of glacial motion have had for their object the assertion of some particular view, rather than the establishment of a complete and sufficient theory founded on well-defined hypotheses and unequivocal definitions, together with a careful comparison of the results of accurate theoretical investigation with those of direct observation. Each of these views has been regarded, in my own opinion improperly, as a *Theory of Glacial Motion.* The Expansion Theory ignored the Sliding Theory, though they were capable of being combined; the latter theory was equally ignored by the Viscous Theory, in which, moreover, instead of the definitions of terms being clear and determinate, no definition of *viscosity* was ever given, though that term designated the fundamental property on which the views advocated by this theory depended. Again, the Regelation Theory is not properly a theory of the motion of glaciers, but a beautiful demonstration of a property of ice, entirely new to us, on which certain peculiarities of the motions of glaciers depend. When we shall have obtained a *Theory of the Motion of Glaciers* which shall command the general assent of philosophers, no qualifying epithet will be required for the word *theory*; it would indeed be inappropriate, as seeming to indicate the continued recognition of some rival theory. If, for instance, it should be hereafter admitted that the sliding of a glacier over its bed and the property of regelation in ice are equally necessary, and, when combined, perfectly sufficient to account for the phenomena of glacial motion, there would be a manifest impropriety, not to say injustice, in selecting either of the terms *sliding* or *regelation* by which to designate this combined theory. I make these remarks because I believe that the preservation of the partial epithets above mentioned has a tendency to prevent our regarding the whole subject in that more general and collective aspect under which it is one of the principal objects of this paper to present it.

This object must necessarily give to the present paper something of the character of a *résumé* of what has hitherto been done, whether it be our purpose to adopt or reject the conclusions of others. There are periods in the history of almost every science when its sound and healthy progress may almost as much demand the refutation of that which is erroneous as the establishment of that which is true. I shall not, however, enter into any review of the past labours of glacialists with respect to exploded theories, but shall only notice those more recent researches and speculations which appear to

me either to demand refutation as erroneous, or admission into any well-founded theory as correct. This treatment of the subject must necessarily lead to a certain repetition as to results already established, and be also of too critical a character, perhaps, with reference to other results in which I may have no confidence. I can only say that, in the present state of glacial theory, such defects must be inherent in any attempt to present it under a more complete and systematic form than it has hitherto, I think, assumed. It is this circumstance, too, which I would especially offer as an apology for the repetition of certain results which I obtained many years ago. Most of them are abstract mathematical results. They are here obtained by a more general analysis of the problem than that formerly employed, and are introduced as essential steps in the development now offered of the theory of the motion of glaciers.

In the first section I shall endeavour to remove the ambiguities which have beset this subject from the want of explicit definitions of certain terms expressing properties of ice on which our theories of glacial motion must essentially depend.

In the second section I shall give a brief statement of the results of experiments which explain the sliding of a mass of ice down a plane of small inclination, with a slow and unaccelerated motion like that of an actual glacier.

In the next section certain propositions are investigated respecting the internal pressures and tensions superinduced within a solid body by external forces which slightly distort it, and produce a small relative displacement of its component particles, or what may be termed a *molecular displacement.*

In the subsequent sections these results are applied to the explanation of crevasses, and to an examination of the theories which have been proposed of the veined structure of glacial ice. Finally, I have shown the importance of the sliding motion in giving efficiency to the internal pressures and tensions to dislocate the glacier, which thus becomes relieved from its internal strain, regaining the continuity of its mass and structure by its property of *regelation* so beautifully exhibited in Dr. TYNDALL'S experiment.

SECTION I.—*Definitions and Explanations of Terms.*

1. The external forms of all bodies in nature may be changed in a greater or less degree, and without producing discontinuity in their mass or destruction of their internal structure, by the action of any external forces, the original or undisturbed form from which the change of form is to be estimated being that which the body would assume if acted on by no external forces whatever. This change of form necessarily implies a change in the relative positions of the component particles of the mass, or a certain greater or smaller amount of *molecular mobility,* or power in the particles of moving *inter se.* We may speak either of the general change of the form of the whole body, or of that which takes place in each of its small elementary portions; it is, in fact, in this latter sense that we are obliged to regard it in any accurate investigations, because the change of form for different elements will usually be different. Change of form in an element may or may not be accompanied by a change of its volume. In the first case it

leads to *cubical* extension or compression; in the latter, merely to extension or compression of the surface and not the volume of the element, which may be called *superficial* extension or compression. These changes of volume and form in any element must be produced by the forces acting on it. Thus we may conceive linear extension alone produced at any interior point of the mass by two equal and opposite tensions acting on two elementary component particles there in the direction of the line joining their centres of gravity, while compression alone would result if those tensions were changed into pressures. In such cases extension or compression would be the result of forces which may be called *direct* or *normal* forces. In the case above mentioned, in which the volume and density of every element of the mass remain unaltered, there can be no such direct normal action as that just mentioned. The action must be perpendicular to the normal, and must therefore be a *transversal* or *tangential* action. There would be no tendency to make the contiguous particles approach to or recede from each other, but to cause the one to *slide* tangentially past the other.

If the body have a structure like that of any hard vitreous or crystalline mass, pressure at any point will tend to break or crush the body, and thus to destroy the continuity of its structure. This tendency will be opposed by the *resisting power* of the substance. The tendency of the direct or normal tension is to separate the contiguous particles, and thus produce a finite fissure, or a discontinuity in the mass. It is resisted by the *normal cohesive power*; and in like manner the transverse or tangential action is resisted by the *tangential cohesion*, or that which prevents the component particles from sliding past each other. Again, when the component particles at any point of a body are relatively displaced, they have always a certain tendency to regain their originally undisturbed position; and the force thus excited, considered with reference to the force of displacement at that point, affords a measure of what is called the *elasticity* of the body. Since the force of restitution may vary from zero to the corresponding force of displacement, the elasticity, when measured by their ratio, may vary from zero to unity.

2. We may now define such terms as *solid*, *plastic*, *viscous*, and the like, with all the accuracy which their definitions admit of. We may call a body emphatically a *solid* body when it possesses the following properties:—(1) small extensibility and compressibility, (2) great power of resistance and great cohesive power, both normal and tangential, and (3) great elasticity. It will thus require a comparatively great force to produce any sensible relative displacement among the constituent molecules of the body: if we conceive the force required to become infinitely great, we arrive at absolute *rigidity* as the limit of solidity. Again, we shall best, perhaps, define *plasticity* or *viscosity*, if we suppose the forces of displacement to be such as to produce only a small transverse or tangential displacement of the constituent particles, *i. e.* a superficial, not a cubical, extension or compression. Then, if the force of restitution bear only an inappreciable ratio to the corresponding force of displacement, *i. e.* if the *tangential elasticity* be not of sensible magnitude, the mass may be emphatically said to be *plastic*. This is the essential condition of what may with strict propriety be termed *plasticity*; it might also

be added that, as bodies are constituted in nature, the force required to produce the original displacement in plastic bodies will be small as compared with that required in solid bodies. *Viscosity* and *semifluidity* are terms which only express similar properties of bodies, but indicating that still smaller forces only are required to produce a given displacement in viscous or semifluid bodies than in plastic ones. The limiting case is that of *perfect fluidity*, in which both the forces of original displacement and those of restitution are indefinitely small. In these latter cases the tangential cohesion is necessarily small, and such also (as bodies are usually constituted) will be the normal cohesion. At the same time the power of resisting compression of volume may be very great, as in fact it is in nearly all masses not technically designated as *elastic masses.* In other words, the *normal* elasticity, with reference to pressure, may be of any magnitude, while the *tangential* elasticity equals zero.

It will be observed that I have here spoken of a body as held in a state of constraint by external forces, but without any kind of dislocation which should destroy its continuity or injure its structure. If, however, the external forces should be sufficiently increased, the structure of a vitreous or crystalline mass, or that of any mass possessing hardness and brittleness, will be destroyed by a pressure greater than its power of resistance can withstand; or the continuity of its mass will be destroyed by any normal tension greater than the normal cohesion, or, again, by any tangential tension greater than the tangential cohesion. The normal tension would then produce an open fissure; and the tangential tension would cause one particle of the mass to slide past another, but without producing any open discontinuity. On the contrary, in a properly plastic or viscous mass there is no definite structure for excessive pressure to destroy; there is no question as to the formation of open fissures; and the characteristic absence of *tangential* elasticity allows of any amount of change in the relative positions of the constituent particles of the mass without breach of its continuity.

It would of course be impossible to draw an exact and determinate line of demarcation between solidity and plasticity, but it is not therefore the less certain that there are bodies which do unequivocally possess the property of solidity, and others which do as unequivocally possess the property of plasticity, according to the definitions I have given of these terms. Solidity and plasticity with respect to numerous cases in nature thus become determinate properties of those aggregates of material particles which we call bodies. Ice, a vitreous or crystalline and brittle mass, which will neither bear any but the smallest extension without breaking, nor more than the smallest compression without being crushed, must be solid, and cannot be plastic, if we are to use those terms as significant of determinate properties of bodies.

3. The advocates of the Viscous Theory would not probably admit the necessity of the above rigorous definition of the term *viscous* in its application to glacial ice. But the defect of that theory has always been in the entire want of any accurate definition of that term. When such a definition was demanded, it was said that glacial ice must be viscous, because a glacier adapted itself to the inequalities of its valley as a viscous

mass would do. This was equivalent to saying that the mass was viscous because it moved in a particular manner, instead of asserting that the mass moved in that particular manner because it was viscous. Now this kind of inversion of the direct enunciation of the proposition is only admissible when there is no other physical cause, than the one assigned, to which it is conceivable that the observed phenomenon should be ascribed. Thus we may assert with perfect conviction, that gravity exists as a property of matter and acts according to a certain law, because the bodies of the solar system move as if such were the case; but the conclusiveness of this inductive proof of the proposition—that "gravity is a property of matter"—rests entirely on our conviction that matter has no other property by which we could equally account for the phenomena of the celestial motions. And so with regard to glaciers. If viscosity were the only conceivable property of ice by which we could possibly account for the observed motion of glaciers, then would the observed phenomena of that motion perfectly convince us of the existence of the property in question. But here the two cases entirely differ, inasmuch as there was no general conviction, nor even a decided probability at the time I allude to, that no physical property of ice could exist besides viscosity which might account for the observed phenomena of a glacier's motion; and at the present time it is proved that there *is* another property of ice by which those phenomena are perfectly accounted for, and the inductive proof of viscosity becomes altogether valueless. Moreover, in the case of universal gravitation, the inductive proof is the only possible one, whereas in glacial motion we are concerned with a property which, in whatever sense the definition of it may be regarded, must be as capable of being rendered patent by experiment in ice, if it exist, as in any other substance.

The answer, then, that was given to the question, what is viscosity? comprised no definition at all of that term. The viscous theory ignored the possibility of the molecular mobility of a glacial mass, united with the preservation of its continuity, being attributable to any other property than that which was designated as *viscosity*, but without giving any exact definition of the term. If it was meant to define by it the property which I have defined by the same term, the theory had a legitimate claim to be considered a *physical* theory, because it assigned a determinate physical property as the cause of certain observed phenomena. In this sense, however, I conceive that it would now be admitted to be entirely disproved by Dr. TYNDALL's experiments, in which the ice exhibits so clearly the property of solidity, and the absence of all indication of plasticity. The hypothesis of viscosity, I imagine, could only have been adopted in the first instance from the apparent absence of any other property of ice which might account equally well for the molecular mobility of the glacial mass.

4. But if the determinate property of viscosity, as I have defined it, be not recognized in ice, what, it will be asked, is really the idea which has been attached to the term plastic or viscous? The question, as I have already intimated, is difficult to answer. Perhaps the best way of doing so is to refer to the "Prefatory Note" to Principal FORBES's 'Occasional Papers' (p. xvi). He there intimates that the expressions "bruising

and re-attachment," and "incipient fissures re-united by time and cohesion," used by him in 1846, are to be regarded as having the same meaning as the expression "fracture and regelation," first introduced into the subject in 1857. Now there is no ambiguity whatever in this latter expression. "Fracture" means the breaking and splitting of the ice regarded as a brittle and crystalline *solid*, and could never be intended to have the slightest reference to viscosity. In fact the expression is altogether inapplicable to any body which can be called viscous, without what I should regard as a violation of scientific language. Still this, it may be said, may be only a want of strict accuracy of expression, rather than of accuracy of conception. But if a notion of cracking and breaking, so foreign to any idea of plasticity, should be admitted, it could not be said that a glacier moved as it is observed to move because it was plastic, but merely that it moved *as if* it were plastic. The true inference from the motion would have been that glacial ice possessed not necessarily real plasticity, a definite property of bodies, but a *quasi-plasticity*, which expresses no determinate property at all, but may consist with many different properties. It merely expresses, in fact, the power of the component elements of the mass of changing to a certain extent their relative positions. But this is not the peculiar property of ice; it is common, indeed, to all bodies exposed to disruptive forces which, as in the case of ice, the cohesive power is unable to withstand. The mass of any other substance, as well as that of a glacier, will then be broken into fragments sufficiently small to allow it to follow the impulses of the external forces acting on it. To say, therefore, that a glacier moves *as if* it were plastic is not to assign to ice any property peculiar to itself, and therefore does not properly constitute a *physical* theory of glacial motion at all.

5. But if we should pass over the difference between true plasticity and that which, as we have pointed out, is merely apparent, there would still remain the great difficulty which was only removed by the experiments of Mr. FARADAY and Dr. TYNDALL. Every one who believed ice to be a solid body, believed as a matter of necessity that a glacier must, on account of the external conditions to which it is subjected, be excessively broken and dislocated in the course of its motion. I was myself one of those who fell into the error of attributing too much influence to the larger and more visible disruptions of the mass; but the great difficulty was in the perfect subsequent reunion of portions which had thus been separated, whether by larger or smaller dislocations. And here it will necessarily be asked whether, in the expression above quoted, "*re-attachment*" and the "*re-union* by time and cohesion" of separated portions when again brought into contact, really mean the same thing as *regelation*? It can only be answered, I think, by saying that, whatever might be the intended meaning of those expressions, they failed to convey to the minds of others the most remote idea of regelation as a property of ice at a particular temperature. No better proof can be given of this than the general conviction which appeared to flash across the mind of every glacialist when he first heard of Dr. TYNDALL'S experiment, that the recognition of the property of instantaneous regelation was a well-marked and important discovery, which had at once completely

removed a great stumbling-block in glacial theory. In fact, the viscous theory assigns no physical cause for the reunion in question. All we could do, before the publication of those experiments, was to infer from the observed facts that ice did possess some property which facilitated the reunion of separate pieces in contact; but this was like the attempt to define viscosity by an appeal to the phenomena which that property was intended to explain.

6. An *imperfect plasticity* in ice has sometimes been spoken of. The fact is, all solid bodies might be said to have an imperfect plasticity, if we chose to admit this vagueness in scientific language, since all are capable of greater or less extension or compression. As to the apparent plasticity inferred from the motion of glacial masses, and arising from the crevicing of the ice, I have explained that it has no relation whatever to real plasticity. Such crevices are the necessary consequences of the external forces acting on the glacier, and are as essential to the theory of regelation as they are unconnected with any property of plasticity.

I proceed, as proposed in my introductory remarks, to explain the *sliding* of glaciers.

Section II.—*Sliding Motion of Glaciers along the bottoms of the Valleys containing them.*

7. The sliding motion of glaciers, as first suggested by De Saussure, seemed to involve some serious difficulties. The inclination of the surfaces along which some of the Alpine glaciers descend is so small (not exceeding, perhaps, in some cases 3° or 4°) as to furnish one very obvious objection; and another was, that if glaciers did thus move at all, it must be by an *accelerated* motion, whereas their real motion was an unaccelerated one. Both these objections were founded on an entirely erroneous conception of the nature of the forces called into action in the sliding of a glacier. They were considered to be analogous to the force of friction in the ordinary case of a body sliding down an inclined plane. But in this latter case the constitution of the sliding body, and of that on the surface of which it slides, is always assumed to be such that the surfaces in contact are not affected by the sliding movement, and then we have the experimental law that the friction is independent of the velocity. Consequently the motion is an *accelerated* one. But the condition just mentioned, respecting the surfaces in contact, will not be satisfied in the case of a glacier, assuming what I shall prove shortly, that its lower surface must necessarily have a temperature not lower than that of freezing, and must consequently be always on the point of disintegration by thawing. Ice then becomes very tender, and the cohesion of its particles at its lower surface becomes insufficient to prevent the descent of a mass of such an enormous weight as that of a glacier, even along planes of the smallest inclination. This was clearly illustrated by a simple experiment which I made some years ago, which also fully explained the unaccelerated motion of a sliding glacier. The details of this experiment will be found in the Cambridge Transactions for 1844 *. It constitutes so fundamental a step in this subject, according to my own views of it, that I would beg permission to give here a brief description of it, and of the results to which it leads.

* They will also be found in the Philosophical Magazine for January 1845.

8. A mass of ice was placed on a flat rough slab of sandstone, so arranged that it could easily be placed at any proposed inclination to the horizon. When the inclination was about 20°, the ice descended with an accelerated motion as in ordinary cases, but at smaller inclinations it descended with a *slow uniform motion*, which, for inclinations not exceeding 9° or 10°, was, *cœteris paribus*, *proportional to the inclination.* The velocity was increased by an *increased weight* of the mass, the area in contact with the plane remaining the same. The motion was due to the melting of the ice in contact with the slab, for when the temperature of the air became below that of freezing, it entirely ceased.

The motion was sensible for inclinations little exceeding half a degree, and, doubtless, would also have been so for smaller inclinations. This shows how small a force was required to move the mass when its lower surface was in a state of disintegration. Let f be a retarding force applied to a mass whose weight $=$W, placed on a plane whose inclination is ι. Then will W sin $\iota - f$ be the moving force along the plane; and since the mass will require only the smallest force to move it, a force f very nearly equal to W sin ι will be necessary to hold the mass at rest. If f be removed, the ice will begin to move with a moving force nearly $=$ W sin ι, the motion being permitted by the liquefaction of successive indefinitely thin layers of ice, but then it will be retarded by the solid mass coming in contact with the surface on which it slides. A melting of another indefinitely thin layer will then take place, and the above process will be repeated, the velocity increasing till the continuous action of the plane on the mass becomes equal to the weight resolved in the direction of the plane. During this time (probably too short to be estimated) the motion will be an accelerated one, but will thenceforward become uniform, the action of the plane becoming equal to the resolved part of the weight along it. The uniform velocity is, in fact, a *terminal* velocity, similar to that of a stone descending in water, when it soon approximates to a nearly uniform motion. The action of the inclined plane on the moving mass, like that of the resistance of the water on the stone, has this property—that, while it is incapable of exerting any but the smallest force to hold the body absolutely at rest, it exerts a retarding force upon it in uniform motion, equal to that of gravity. The objections above mentioned against the sliding of glaciers have arisen from an entire misconception of this kind of mechanical action.

9. At the period when the preceding experiment was published, I was disposed to think that the greater part of the observed motion of the surface of a glacier was due to the general motion, by sliding, of the whole mass, while it was contended by other glacialists that it was principally due to an excess of the velocity of the upper strata of the mass over that of its lower strata, due to a gradual change of form of the whole mass, and that there might in fact be no sliding movement at all. In recognizing that both these causes might be *veræ causæ* *, I urged the necessity of deciding on their relative claims by actual observations, which should determine the velocities of the upper and lower surfaces of a glacier at some point where the lower one was accessible. Observa-

* Philosophical Magazine and Journal for February 1845.

tions for this purpose were made by Principal FORBES in 1846, at the bottom of the Glacier des Bois at Chamouni. He found that the velocity of a point on the surface, at the height of 143 feet above the bed of the glacier, was to that of a point 8 feet above the bed, in the ratio of nearly 16 : 10 *, whence it follows that, if we divide the whole velocity of the surface into eight parts, five of them will be due to the motion of the bottom of the glacier, and three to that change of form of the mass by virtue of which its higher move faster than its lower portions. Dr. TYNDALL has also made similar observations on the flank of the Mer de Glace †, from which it appears that the motion of the upper was there rather more than twice that of the lower surface. These observations may or may not determine approximately the average ratio between the velocities of the upper and lower surfaces of a glacier; but they leave no doubt as to the fact of the sliding movement. Again, it is observed that existing glacial valleys, and those which are believed to have been such in former times, always indicate, by their smoothed and striated rocks, the sliding movements of the glaciers they formerly contained. In fact, few glacialists at present, I imagine, will doubt the existence of this sliding motion, or that it forms a considerable portion of the whole motion of a glacier; and I believe that the experiments above described afford an adequate explanation of the cause and character of that motion. I insist on this more particularly because the explanation has been singularly ignored and misunderstood. The non-applicability of the experiments has been asserted, because the sliding mass was not obstructed in its motion by lateral obstacles, like a glacier, whereas, in fact, they had no concern with lateral obstacles, being merely intended to explain the action of the bed of the valley on the superincumbent glacier. The irregularity of the sides introduces, as we shall see very shortly, no difficulty or ambiguity into my views of the subject. I may also state that, several years after my experiments and his own observation above stated were published, Principal FORBES repeats his objection of the difficulty of conceiving the possibility of the motion of sliding glaciers being unaccelerated, whereas every one now acknowledges that they do slide, and knows that their motion is unaccelerated ‡. M. AGASSIZ, on the contrary, after repeating the experiments, allows that the results remove the great difficulties of admitting the sliding motion of glaciers §. This kind of motion, as we have seen, depends very much on the temperature of the lower surface of the glacier being always equal to the freezing-temperature. That such must always be the case I proceed to show. All observations indicate that it is so; but still, since few direct observations can be made on this point, it may be well to show that it follows from the temperature of the earth, and the nature and conductivity of ice, that the temperature of the lower surface of a glacier must be that above mentioned.

* Occasional Papers, p. 175.

† Glaciers of the Alps, p. 289.

‡ "The main objection, however, is this, that a sliding motion of the kind supposed, if it commence must be accelerated by gravity, and the glacier must slide from its bed in an avalanche. The small slope of most glacier-valleys, and the extreme irregularity of their bounding walls, are also great objections to the hypothesis."—Occasional Papers; also published in 1855 in the 'Encyclopædia Britannica.'

§ Système Glaciaire, p. 568.

10. *Interior Temperature of Glaciers.*—There are two obvious causes by which the temperature in the interior of a glacier may be affected—(1) conduction of heat from the superficies of the mass according to the ordinary laws of conduction through solid bodies, and (2) infiltration of water from the upper surface. We may consider separately the operation of these causes, with the view of determining whether the lower surface of the mass is permanently at the temperature of freezing, as it must be to render the preceding experiments strictly applicable to account for the continuous and unaccelerated motion of the glacier. To investigate the effect of the first cause, let us conceive the whole surface of the earth covered with a superficial crust of ice. The temperature of this crust will be subject to periodical annual variations to a certain depth, which will depend on the annual variations of the superficial layer of the icy crust, and the conductivity of the ice. It may be considered, for our immediate purpose, sensibly the same as the temperature of the glacier itself, at all points where the glacier and our imaginary crust of ice coincide. Now the surperficial changes of temperature in the crust of ice subject to conditions similar to those of a glacier, would be much less than those which the actual rocky crust of the earth experiences; for the temperature of the ice could never rise above 32° Fahr. in the hottest summer, nor in the coldest winter could it probably fall many degrees below that temperature, on account of the covering of snow like that which on all glaciers protects their outer surface against the effect of low winter temperature. Again, the depth through which these oscillations of temperature would be perceptible, would, *cæteris paribus*, be comparatively small if the conductive power of the mass should be so. I am not aware of any experimental determination of the value of this power for ice; but that substance is known to be a very bad conductor, and probably worse than the average of the rocks which form the outer crust of the earth. For both these reasons, then, the depth of oscillatory annual temperature would be much smaller than it is found to be within the actual crust of the globe, under the same external climatal conditions. Now in the earth's crust, and in our own latitudes, this depth may be approximately estimated at 70 or 80 feet, according to the nature of the upper strata. I should therefore deem it probable that the variations of external temperature in a crust of ice like that above supposed, or therefore in an ordinary glacier, would not exceed at most perhaps some 30 feet.

Again, let us consider the probable *mean* annual superficial temperature of our hypothetical icy crust, or of a glacier of ordinary dimensions. The actual temperature could never rise, as above remarked, above 32° Fahr., and would never sink many degrees below that temperature. M. AGASSIZ has left us the only reliable observations on this subject*. He buried a self-registering thermometer in the glacier of the Aar, at the depth of 2·1 metres. It was taken up two years afterwards, and was found to have registered a minimum temperature of $-2^{\circ}{\cdot}1$ (C.) $=28^{\circ}{\cdot}22$ (Fahr.). Thus the mean temperature for the winter was probably not less than 30° (Fahr.), and that for the

* Système Glaciaire, p. 425.

whole year might perhaps not much fall short of 31°(Fahr.)*. If the conductive power of ice were equal to that of the earth's crust, the mean temperature would increase 1° (Fahr.) in descending some 60 or 70 feet; and therefore, on account of the smaller conductivity of ice, it would probably, in the case of a glacier, rise to 32° (Fahr.) at a depth of some 30 or 40 feet. This would hold, it should be observed, on the supposition that the substance below this depth should be capable, like the matter of the earth's crust, of taking any temperature higher than 32° (Fahr.). This higher temperature would be acquired, as in the actual case of the earth, by the flow of heat from the earth's interior. But in the case of a glacier this heat will be expended in melting the lower stratum of ice instead of communicating a higher temperature to the whole mass. Consequently, if the thickness of the glacier exceed some 30 or 40 feet (a depth at which, as above shown, the temperature will be invariable), the temperature of the lower surface will be constant and equal to 32° (Fahr.).

The temperature at any proposed point (P) of the interior of a glacier, at a depth greater than that estimated above at some 30 or 40 feet, will always be constant and less than 32° (Fahr.), provided the mean annual temperature of the external surface of the glacier be so. To find this constant temperature at P, take for the temperature of the upper surface its mean annual temperature. Let it $=T^{\circ}$ (Fahr.). Also let $a=$thickness of the glacier, $x=$distance, from the upper surface, of the proposed point, and t its required temperature. Then shall we have, according to the laws of conduction of heat,

$$\frac{t-T^{\circ}}{32^{\circ}-T^{\circ}}=\frac{x}{a},$$

the difference of temperatures, as is well known, being approximately proportional to the distances from the upper surface. Hence

$$t=T^{\circ}+\frac{x}{a}(32^{\circ}-T^{\circ}),$$

which shows that t must always be greater than T°; it must also be less than 32° (Fahr.), and must therefore lie between those quantities. Consequently, since $32^{\circ}-T^{\circ}$ is small for the Alpine glaciers, their internal temperature must be nearly uniform, but always a little below 32° (Fahr.), supposing it to depend only on the process of conduction.

But the process of infiltration will tend to raise the internal temperature more nearly to 32° Fahr.; for since the infiltrated water will have that temperature, it will constantly tend to heighten the temperature of the mass through which it passes till it rise to 32° (Fahr.), and never to lower it. This water must thus bring to the glacier (a mass of lower temperature than itself) a continual accession of heat, which it can only lose again by conduction through the upper surface during the winter; and this loss will be restricted to that small depth beyond which the annual variations of temperature cannot extend. For all points at greater depth infiltration must ultimately raise the temperature to 32° (Fahr.).

* M. AGASSIZ states that the temperature given by his experiment might possibly be somewhat too high. There is no probability, however, that the error would be sufficient to affect the reasoning in the text.

Hence, then, considering the combined operation of conduction and infiltration, it appears that, to the depth of perhaps 30 feet, the *interior* temperature of a glacier will be 32° (Fahr.) during the summer portion of the year, but will be rather lower than 32° (Fahr.) during the winter. For all deeper parts of the glacier it will be invariably equal to 32° (Fahr.).

These results are in exact accordance with the careful observations made by M. AGASSIZ, which have already been partially referred to. Besides the winter observations above mentioned, he also observed the temperatures in the month of July, at depths of from 3 to 5 metres, at 30, and at 60 metres. These temperatures were all exactly 32° (Fahr.) during the fortnight they were observed, with the exception of one or two very small and manifestly accidental variations in the more superficial observations.

11. It follows from the preceding articles that the temperature at the lower surface must always be the freezing-temperature, *i. e.* the ice there must be in that state in which the mutual cohesion of its constituent particles is less than in any other state. It does not follow that the glacier would not slide if the temperature of its lower surface were less than 32° (Fahr.); but that temperature is the most favourable for the motion of the glacier, because the most favourable to the disintegration of its lower surface, and the immediate conversion of the ice which forms it into water. It should be remembered, too, that it was one of the results of my experiment, that, *cæteris paribus*, the motion was increased by increasing the weight of the mass; *i. e.*, the cohesive power of the ice at the bottom of the glacier will be the more rapidly overcome by increasing the depth of the mass, the area of its base being unchanged. Consequently the tendency of a glacier to descend down its bed would be indefinitely greater, *cæteris paribus*, than that of our experimental lump of ice down its plane. It is this enormously increased tendency that enables the mass of a glacier to overcome the resistance arising from the inequalities of the sides and bottom of its valley. We shall explain in the sequel the prodigious force which may thus be exerted, and the corresponding internal tensions which would thus be produced by it. These tensions overcome the cohesion of the mass, the ice breaks, and the glacier obtains more freedom of motion than it could have in its state of greater compactness and continuity. The tendency to the sliding motion we are considering will manifestly be greater in the axial than in the marginal parts of the mass. It is there, especially, that the depth must be the greatest, and the distance from opposing lateral objects is likewise greatest; and, it may be added, the subglacial currents, by which the sliding will undoubtedly be more or less facilitated, will be generally greatest along the central parts of the valley.

Hence, then, it follows that, so far as the motion of a glacier depends on the *sliding* we have been considering, the velocity of its axial portions will generally be considerably greater than that of its marginal portions. This constitutes the most distinctive and important character of the observed motion of a glacier.

Still, though the sliding motion was perfectly consistent with this observed general character of glacial motion, it was not sufficient to account for several striking pheno-

mena attendant upon it. The formation of crevasses was a necessary consequence of the forces acting on the glacier, and the conditions to which it was subjected; but no reason was thus assigned why such crevasses should be obliterated again, as they were frequently observed to be, and the continuity of the mass perfectly restored. Moreover, it became evident from more accurate and detailed observation, that the continuity of many parts of the general mass was preserved in a degree apparently inconsistent with the change of form to which a mass so crystalline and brittle as glacial ice did manifestly submit. It was to meet this difficulty that Principal FORBES was led to the hypothesis of the *viscosity* or *plasticity* of glacial ice. I have already explained my objection to the vagueness with which these terms appear to me to have been used, and to the total want of all experimental proof of any property in ice which could be so designated with accuracy, or with a due regard to the propriety of scientific language. Difficulties of this kind always remained on the minds of certain glacialists, till the experiments of Mr. FARADAY and Dr. TYNDALL at once explained to us that *regelation*, and not *viscosity*, was the real property of ice required for the completion of our general theory of glacial motion. This property of regelation belongs essentially to *solid* bodies, and in treating glacial masses as bodies possessing the property of regelation, we must necessarily treat them as *solid*. As such I consider them in the following investigations, the object of which is to ascertain, as far as we are able, the internal pressures and tensions to which glaciers are subjected, and the phenomena which may result from them, more especially those connected with the veined structure of glacial ice, and the formation of crevasses.

12. Before I proceed to these investigations, I would here remark that the importance of a distinct conception of the properties indicated by the terms viscosity or plasticity on the one hand, and solidity on the other, will be at once apparent if we consider the difference between the mechanical problems presented to us in the motion of glaciers, according as we conceive them to be typified by a viscous or solid mass. In the first case we have to determine the continuous motion of a mass the component particles of which move with different velocities, but without destroying its continuity. The most simple, and the limiting case, would be that in which the tangential action of contiguous particles on each other should vanish. The mass would then become a fluid mass. But even in this case we can do little by accurate mathematical investigation, and still less in the case in which the mass is viscous. Consequently, the objection against any attempt at a mathematical solution of the problem of glacial motion, founded on our ignorance of the motions of viscous masses, is perfectly valid, so long as we treat glacial ice as viscous according to our definition of that term. But this same objection has been urged against all attempts to apply accurate mathematical processes to the problem, in its complete or partial solution, under the supposition of ice having the property of solidity. The complete solution of the problem would undoubtedly be far more difficult for a solid than for a viscous mass; for it would involve conditions depending on innumerable discontinuities in the mass, resulting from its motion. All that can be attempted is a

partial solution of the problem, in which the dynamical difficulties are evaded. It has been already explained that when a solid body is acted on by external forces, it generally becomes distorted in form and changed in volume. If the forces be insufficient to overcome the cohesion of the mass and to dislocate it, they will of course continue to maintain the body in its state of constraint and distortion, and thus to produce, at different points of its interior, pressures and tensions varying both in direction and intensity. The immediate object of the first part of the investigations contained in the following pages, is the proof of certain propositions respecting these internal pressures and tensions, and the phenomena resulting from them. The problem thus considered is not a dynamical, but a statical one, in which certain results are attainable with the same accuracy as in the simplest mechanical problems; and such are the only results with which we are directly concerned in our present researches. If the distorting forces be sufficiently increased, the mass will be torn or crushed, as already stated, and will then move according to the new conditions imposed upon it in its state of dislocation. This constitutes the dynamical part of the problem, but, it must be recollected, it does not enter at all into the mathematical part of our own investigations. I have thought it necessary to point out this distinction, lest any vague objection resting on an imperfect or erroneous conception of the problem before us should exercise an undue influence on the mind of the reader.

SECTION III.—*On the Pressures and Tensions at any point of a Solid Mass held in a position of constraint by external forces.*

13. We may now proceed to the consideration of the general problem, the object of which is to investigate the nature of the internal pressures and tensions at any proposed point of a solid mass subjected to the action of impressed forces which slightly distort it from the form it would assume when acted on by no external forces at all. It will be recollected that these forces are supposed insufficient to destroy the continuity of the mass. They maintain it in its distorted form, and must therefore be in *equilibrium* with the internal forces arising from the cohesive power of the mass.

To explain the nature of the distortion produced in any small element of the mass, let us denote by s the area of an indefinitely small plane surface passing through any point (P). Generally there will be an action between the particles (M) on one side of our small plane, and those (M′) on the opposite side. Since s is indefinitely small, we may represent by f the whole action of M on M′, and suppose its direction to make an angle δ with the normal to the plane s. Then will

$$f\cos\delta \text{ and } f\sin\delta$$

be the normal and tangential actions respectively of M on M′; and

$$-f\cos\delta \text{ and } -f\sin\delta$$

will be the corresponding actions of M′ on M. If the normal force be a *pressure*, it will

only tend to preserve the particles on opposite sides of s in contact, but if it be a *tension*, it will tend to separate those particles by motions parallel to the normal. Also the tangential forces $f \sin \delta$ and $-f \sin \delta$ will always tend to separate two particles on opposite sides of s and in contact, by making them move in opposite directions parallel to the plane. If we conceive s to revolve about P as a fixed point, and thus to assume all possible angular positions, the forces $f \cos \delta$ and $f \sin \delta$ will vary with the angular position of the plane, in certain positions of which they will assume their maximum and minimum values. The determination of these positions is one immediate object of the problem, with the view of determining the effect of the distorting forces on the form of each element of the mass, and thence, if the problem were completely soluble, the distortion of the whole mass. But before proceeding further, we may explain more explicitly what will be the kind of distortion to which every element of any solid body will be subjected under the action of distorting forces. Let us take a small rectangular parallelopiped as the element of the body while unconstrained by such forces. The normal forces acting on opposite sides of the element will manifestly form three pairs of equal* and opposite forces, each force of each pair acting in a direction opposite to the other force of that same pair, and thus producing compression or extension of the element according to the directions in which they act. Again, it is manifest, from what has been said respecting the small plane s, that the tangential force on any one side of the elementary parallelopiped will be equal to that on the opposite side, but will act in the opposite direction, thus tending to *twist* the element from its original rectangular form into an oblique-angled parallelopiped. Hence the primitive undistorted element will be compressed or extended according to circumstances, and will always (unless the forces acting on it be entirely normal) be twisted so as to destroy its rectangularity. In the final application of the results obtained from this our typical problem, we shall have to deduce the manner in which the continuity of the glacial mass will be destroyed when the power of resistance of the ice is no longer sufficient to equilibrate the distorting forces acting on it, and also to consider the phenomena which may result from such breach of continuity.

With respect to the solution of our abstract problem, I have little to add to that which I gave in the Transactions of the Cambridge Philosophical Society for the year 1847, and I might be content merely to refer to that solution for the results. In doing so, however, it would be necessary to give a somewhat complicated notation, and certain explanations in such detail that the space thus occupied would not differ materially from that required for the mathematical analysis of the first part of the general problem. By giving this analysis here, considerable trouble of reference will be avoided. I would request permission, therefore, to repeat a part of what appeared in the Transactions above referred to. The quotation includes the following articles, from the 14th to the 17th inclusive:—

* Omitting small quantities of a certain order, which it is not necessary in this general explanation to take into account.

14. "Taking any point (P) of the mass, let it be made the origin of coordinates xyz. Let the small plane s be conceived as above to pass through P, and let the forces upon it when in the positions specified below be denoted as follows, all being referred to a unit of surface.

"(1) When a perpendicular to the plane coincides with the axis of x, let

$$\text{The } \textit{normal force} = \mathrm{A}\text{; the } \textit{tangential force} = \begin{cases} \mathrm{B}' \text{ parallel to } y, \\ \mathrm{C}' \quad \text{,,} \qquad z. \end{cases}$$

"(2) When a perpendicular to the plane coincides with the axis of y, let

$$\text{The } \textit{normal force} = \mathrm{B}\text{; the } \textit{tangential force} = \begin{cases} \mathrm{C}'' \text{ parallel to } z, \\ \mathrm{A}' \quad \text{,,} \qquad x. \end{cases}$$

"(3) When a perpendicular to the plane coincides with the axis of z, let

$$\text{The } \textit{normal force} = \mathrm{C}\text{; the } \textit{tangential force} = \begin{cases} \mathrm{A}'' \text{ parallel to } x, \\ \mathrm{B}'' \quad \text{,,} \qquad y. \end{cases}$$

"Between the six accented quantities there are three essential relations, which are easily found. On the three coordinate axes at P, construct an indefinitely small parallelopiped whose edges are δx, δy, δz. The six equations of equilibrium of this element will express the conditions that the sums of all the resolved parts of the forces parallel to the coordinate axes shall respectively be equal to zero; and that the moments of the forces with reference to these axes shall also severally be equal to zero. Let us take the three latter conditions, lines through the centre of gravity of the element and parallel to the coordinate axes being taken for the axes of the component couples. The tangential force parallel to the axis of x on the side $\delta x . \delta z$ being A$'$, that on the opposite side will be $-\left(\mathrm{A}'+\frac{d\mathrm{A}'}{dy}\cdot\delta y\right)$; and the couple resulting from these forces about the axis parallel to z, will be

$$\mathrm{A}'\delta x\delta z . \frac{\delta y}{2} + \left(\mathrm{A}'+\frac{d\mathrm{A}'}{dy}\delta y\right)\delta x\delta z . \frac{\delta y}{2};$$

or, omitting small terms of the fourth order,

$$\mathrm{A}'\delta x\delta y\delta z.$$

"Similarly, the couple arising from the forces B$'$ and $\mathrm{B}'+\frac{d\mathrm{B}'}{dx}\delta x$ about the same axis parallel to z, will be

$$-\mathrm{B}'\delta x\delta y\delta z.$$

"Also the moments of the normal forces A, B, C, with reference to the above-mentioned axes, will be zero, always omitting small quantities of the fourth order. Consequently the whole moment of the forces on the parallelopiped with reference to the axis parallel to that of z, will be

$$(\mathrm{A}'-\mathrm{B}')\delta x\delta y\delta z,$$

which must = zero by the conditions of equilibrium; and therefore

$$\mathrm{A}'=\mathrm{B}'.$$

In exactly the same way we find, by taking the moments with reference to the axes parallel respectively to those of y and x,

$$A''=C',$$
$$B''=C'''.$$

By means of these three relations the six accented quantities are reduced to three independent quantities.

15. "Let us now conceive a plane to meet the three coordinate planes so as to form with them a tetrahedron, whose vertex is at the origin P. Suppose the exterior normals to the three faces formed by the coordinate planes to point respectively towards the positive directions of x, y, and z; and let α, β, γ be the angles which the normal to the base of the tetrahedron makes with the coordinate axes of x, y, z. Also, let s denote the area of the base, and s', s'', s''' the areas of the sides of the tetrahedron perpendicular respectively to the axes of x, y, z, all these quantities being indefinitely small.

"Again, let ps denote the whole resultant force on s, and let λ, μ, ν be the angles which its direction makes with lines parallel to the axes x, y, z, this direction being exterior to the tetrahedron. Then in order that the tetrahedron may be in equilibrium, we must have

$$ps.\cos\lambda=As'+A's''+A''s''',$$
$$ps.\cos\mu=Bs''+B's'+B''s''',$$
$$ps.\cos\nu=Cs'''+C's'+C''s'';$$

but

$$\frac{s'}{s}=\cos\alpha,\quad \frac{s''}{s}=\cos\beta,\quad \frac{s'''}{s}=\cos\gamma;$$

making these substitutions, and also putting

$$B''=C''=D,$$
$$A''=C'=E,$$
$$A'=B'=F,$$

we shall have

$$\left.\begin{aligned} p.\cos\lambda &=A\cos\alpha+F\cos\beta+E\cos\gamma,\\ p.\cos\mu &=B\cos\beta+F\cos\alpha+D\cos\gamma,\\ p.\cos\nu &=C\cos\gamma+E\cos\alpha+D\cos\beta,\end{aligned}\right\}\quad \ldots\ldots\ldots \quad (a.),$$

formulæ in which the notation agrees with that of M. Cauchy*.

16. "If δ denote the angle between the direction of p and the normal to s, we shall have $p.\cos\delta$ for the whole *normal* force acting on the area s in a direction exterior to the tetrahedron, and $p.\sin\delta$ the whole *tangential* force acting on the same area. Our first object will be to determine α, β, and γ, or the position of the base s of the tetrahedron, so that the normal action upon it, $p\cos\delta$, shall be a maximum. We shall afterwards have a similar investigation with reference to the tangential force $p.\sin\delta$.

* Exercices de Mathématiques, vol. ii. p. 48.

"We have

$$\cos\delta = \cos\lambda\cos\alpha + \cos\mu\cos\beta + \cos\nu\cos\gamma,$$

whence we immediately obtain

$$p\cos\delta = A\cos^2\alpha + B\cos^2\beta + C\cos^2\gamma + 2D\cos\beta\cos\gamma + 2E\cos\alpha\cos\gamma + 2F\cos\alpha\cos\beta; \quad (1.)$$

and since

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1, \quad \dots \dots \quad (2.)$$

we have (L being an arbitrary multiplier),

$$(A+L)\cos^2\alpha + (B+L)\cos^2\beta + (C+L)\cos^2\gamma + 2D\cos\beta\cos\gamma + 2E\cos\alpha\cos\gamma + 2F\cos\alpha\cos\beta = max.$$

Hence

$$\left.\begin{aligned} \{(A+L)\cos\alpha + E\cos\gamma + F\cos\beta\}\sin\alpha = 0, \\ \{(B+L)\cos\beta + D\cos\gamma + F\cos\alpha\}\sin\beta = 0, \\ \{(C+L)\cos\gamma + D\cos\beta + E\cos\alpha\}\sin\gamma = 0. \end{aligned}\right\} \quad \dots \dots \quad (b.)$$

"To satisfy these equations together with (2.), we must equate the first brackets to zero. We thus have four equations from which L may be eliminated, and α, β, and γ determined.

"If we multiply the first factors on the left-hand sides of equations (*b.*) by $\cos\alpha$, $\cos\beta$, $\cos\gamma$ respectively, and add them together, we have, by virtue of equations (*a.*),

$$L = -p\cos\delta;$$

and substituting for L in equations (*b.*), we have

$$\begin{aligned} p\cos\delta\cos\alpha &= A\cos\alpha + F\cos\beta + E\cos\gamma \\ &= p\cos\lambda \end{aligned}$$

by the first of equations (*a.*); and therefore

$$\cos\delta\cos\alpha = \cos\lambda,$$

and similarly

$$\cos\delta\cos\beta = \cos\mu,$$
$$\cos\delta\cos\gamma = \cos\nu,$$

whence

$$\cos^2\delta = 1$$
$$\delta = 0,$$

which shows that when the resultant force p is a maximum or minimum, its direction coincides with that of the normal to the plane *s*. Consequently, also, the tangential force, $p\sin\delta$, then becomes = zero.

"This value of δ gives $L = -p$; and substituting for L in equations (*b.*), we have

$$\left.\begin{aligned} (A-p)\cos\alpha + F\cos\beta + E\cos\gamma = 0, \\ F\cos\alpha + (B-p)\cos\beta + D\cos\gamma = 0, \\ E\cos\alpha + D\cos\beta + (C-p)\cos\gamma = 0; \end{aligned}\right\} \quad \dots \dots \quad (c.)$$

and eliminating $\cos\alpha$, $\cos\beta$, and $\cos\gamma$ by cross multiplication, we obtain

$$(A-p)(B-p)(C-p) - D^2(A-p) - E^2(B-p) - F^2(C-p) + 2DEF = 0. \quad \dots \quad (3.)$$

"If we take the three values of p deducible from this equation, and substitute them successively in equations (c.), those equations combined with (2.) will give three distinct systems of values for $\cos\alpha$, $\cos\beta$, and $\cos\gamma$, belonging (as is well known) to three lines perpendicular to each other.

17. "Hence it follows that there is at every point (P) of a continuous solid mass under extension or compression, a system of three rectangular axes, such that if the small plane s at P be so placed that its normal shall coincide with one of those axes, the whole resultant action on s shall be *normal* to it, the tangential action upon it being then equal to zero. These three axes are called *the axes of principal pressure or tension* with reference to the point P.

"Of the three values of p in these directions, though they all satisfy the conditions of maximum or minimum, one is a maximum, another a minimum, and the third is neither an absolute maximum nor an absolute minimum. This is best explained, perhaps, by converting equation (1.), as CAUCHY has done, into an equation to a surface of the second order, by putting

$$p\cos\delta=\frac{1}{r^2},\quad r\cos\alpha=x,\quad r\cos\beta=y,\quad r\cos\gamma=z.$$

The inverse of the square of any radius vector will be a measure of the normal action through P perpendicular to this radius vector, the axes of this surface of the second order coinciding with the axes of principal tension or pressure. Of the three principal axes of this surface, the directions of the greatest and least will manifestly coincide with those of minimum and maximum tension; but though the tension in the direction of the mean axis of the above surface satisfies the two conditions $\frac{d.p\cos\delta}{d\alpha}=0$ and $\frac{d.p\cos\delta}{d\beta}=0$, it satisfies the one because it is maximum with respect to α, and a minimum with respect to β, or the converse, as the mean axis of an ellipsoid is a maximum in one principal section of the surface, and a minimum in the other."

18. The object of the second part of this investigation* is to determine the angular positions of the small plane (s) passing through P, so that the tangential force acting upon it shall be greatest, *i. e.* that $p\sin\delta$ may be a maximum. Our formulæ will be much simplified by taking the axes of principal tension or pressure as the coordinate axes. In this case we shall have

$$D=0,\quad E=0,\quad F=0;$$

and if A_1, B_1, C_1 now represent the principal tensions at the proposed point, and α_1, β_1, γ_1 be the values of α, β, γ referred to these new axes, equations (a.) will give

$$p^2=A_1^2\cos^2\alpha_1+B_1^2\cos^2\beta_1+C_1^2\cos^2\gamma_1,$$

and equation (1.) gives

$$p\,.\cos\delta=A_1\cos^2\alpha_1+B_1\cos^2\beta_1+C_1\cos^2\gamma_1.$$

* A solution of the problem above investigated was also given by M. CAUCHY, in his 'Exercices de Mathématiques' (vol. ii. p. 48). The solution of the problem in this second part of the investigation has only been given, I believe, by myself.

Hence we have (if $p \sin\delta = \mathrm{T}$)

$$\mathrm{T}^2 = p^2 \sin^2\delta = \mathrm{A}_1^2 \cos^2\alpha_1 + \mathrm{B}_1^2 \cos^2\beta_1 + \mathrm{C}_1^2 \cos^2\gamma_1 - (\mathrm{A}_1 \cos^2\alpha_1 + \mathrm{B}_1 \cos^2\beta_1 + \mathrm{C}_1 \cos^2\gamma_1)^2,$$

which is to be a maximum subject to the condition

$$\cos^2\alpha_1 + \cos^2\beta_1 + \cos^2\gamma_1 = 1.$$

The solution of this problem is longer and more complicated than that of the problem solved in the first part of the investigation. It will here be sufficient for my purpose to state the results, and, for the analytical solution, to refer the reader to the volume of the Cambridge Transactions above quoted. The results are as follows:—

α_1, β_1, and γ_1 being the angles which define the position of the small plane s (art. 13), the analytical conditions for the tangential force (T, or $p \sin\delta$) upon it being a maximum or minimum, are satisfied by the following three systems of contemporaneous values:—

$$\left.\begin{array}{lll} (1) & \alpha_1 = 90^\circ, & \beta_1 = \gamma_1 = \pm 45^\circ, \\ (2) & \beta_1 = 90, & \gamma_1 = \alpha_1 = \pm 45, \\ (3) & \gamma_1 = 90, & \alpha_1 = \beta_1 = \pm 45; \end{array}\right\} \quad \ldots \ldots \quad (d.)$$

and if T_1, T_2, and T_3 be the values of T corresponding respectively to these systems of values of α_1, β_1, and γ_1, we have

$$\mathrm{T}_1^2 = \tfrac{1}{4}(\mathrm{B}_1 - \mathrm{C}_1)^2, \quad \mathrm{T}_2^2 = \tfrac{1}{4}(\mathrm{A}_1 - \mathrm{C}_1)^2, \quad \mathrm{T}_3^2 = \tfrac{1}{4}(\mathrm{A}_1 - \mathrm{B}_1)^2.$$

If A_1, B_1, C_1 be taken, as they always may be, in order of magnitude, T_2 will manifestly be the greatest of these values of T. It is in fact, as shown in the memoir referred to, the only value which satisfies all the conditions of a maximum. The corresponding values of α_1, β_1, and γ_1, which determine the corresponding position of the plane s, are those given by the second system of (*d.*). Now β_1 is the angle between the normal to s and the axis of y; and since it is here $=90^\circ$, the normal to s must lie in the plane of xz, and the plane s itself must pass through the axis of y. Moreover, since the corresponding values of α_1 and γ_1 are each $\pm 45^\circ$, this plane may have two positions, in both of which it bisects the angle between the coordinate planes of xy and yz, these positions being on opposite sides of the plane of yz. These considerations, however, only determine the position of the plane in which the maximum tangential force (T_2) acts; they do not determine the linear direction of the force in that plane. It is easily shown that it is perpendicular to the axis of y*. Since we have here $\mathrm{D}=0$, $\mathrm{E}=0$, and $\mathrm{F}=0$ (art. 15), we have from equations (*a.*),

$$\begin{aligned} p \cos\lambda &= \mathrm{A}_1 \cos\alpha_1, \\ p \cos\mu &= \mathrm{B}_1 \cos\beta_1, \\ p \cos\nu &= \mathrm{C}_1 \cos\gamma_1, \end{aligned}$$

p being the whole resultant force on the small plane s, referred to a unit of surface, and

* No proof of this is given in my memoir above referred to in the Cambridge Transactions.

λ, μ, ν being the angles which its direction makes with our present coordinate axes of x, y, z respectively. Hence

$$p^2 \quad = A_1^2 \cos^2 \alpha_1 + B_1^2 \cos^2 \beta_1 + C_1 \cos^2 \gamma_1,$$

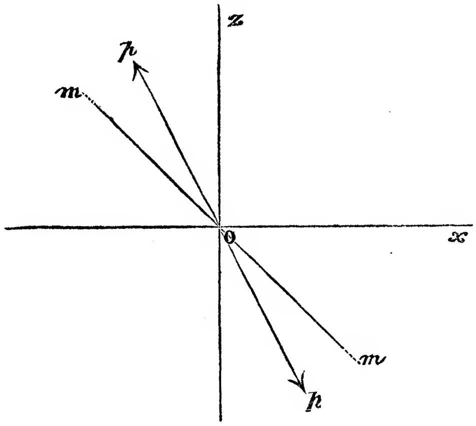
$$\cos \lambda = \frac{A_1 \cos \alpha_1}{\sqrt{A_1^2 \cos^2 \alpha_1 + B_1^2 \cos^2 \beta_1 + C_1^2 \cos^2 \gamma_1}},$$

$$\cos \mu = \frac{B_1 \cos \beta_1}{\sqrt{A_1^2 \cos^2 \alpha_1 + B_1^2 \cos^2 \beta_1 + C_1^2 \cos^2 \gamma_1}},$$

$$\cos \nu = \frac{C_1 \cos \gamma_1}{\sqrt{A_1^2 \cos^2 \alpha_1 + B_1^2 \cos^2 \beta_1 + C_1^2 \cos^2 \gamma_1}}.$$

But $\beta_1 = 90°$ in the case before us, and therefore μ must $= 90°$, *i. e.* the direction of the *whole resultant force* (p) on the small plane s must be perpendicular to the axis of y, and must lie in the plane of xz. Thus, the plane of the paper representing that of xz (fig. 1), the direction Op in that plane may represent the direction of p; and if Om bisect the angle between the coordinate axes x and z, that line will be the trace of a plane on xz coinciding with the plane of s, and therefore perpendicular to that of xz. Consequently, if we resolve the whole force p normally and tangentially with reference to the plane s, the tangential part will evidently coincide with Om. But, from the particular values of α_1, β_1, and γ_1, this tangential force must necessarily be the *maximum* tangential force T_2. Consequently, if we call the axis of y the *axis of mean principal tension or pressure*, the direction of the maximum tangential force (T_2) will be perpendicular to this mean axis, and will bisect the angle between the other two axes of principal tension.

Fig. 1.



19. Hence, in our first problem, the equations (2.) and (c.) determine α, β, γ, and p. The cubic for finding p, which is deduced from them, shows that there are three values of that quantity, the three principal tensions, whose directions are defined by corresponding values of α, β, and γ, and which are at right angles to each other. These quantities being known, the value of T_2, the maximum tangential force at the proposed point, and the direction in which it acts, are immediately determinable from the results of the second part of the problem as given above. To do this we must first determine the three systems of values of α, β, and γ which have been denoted by α_1, β_1, and γ_1, and which determine the positions of the axes of principal tension; and also the values of the three principal tensions which have been above denoted by A_1, B_1, and C_1. For the greater simplicity we then take these axes of principal tension for those of x, y, and z. If A_1, B_1, and C_1 be in order of algebraical magnitude, the axis of y will be the mean axis. If A_1 be a pressure and therefore negative, the proper order will be B_1, C_1, $-A_1$,

and the axis of z will become the mean axis. Other cases must be treated in a similar manner, always preserving the order of algebraical magnitudes for determining the mean axis. The line of greatest tangential action is always perpendicular to it, and bisects the angle between the other two axes of maximum and minimum tension. In determining the magnitude T_2, we must take the same precaution, in arranging the principal pressures in their proper order of magnitude, to determine which are algebraically the greatest and least. Thus in the above case, where the order is B_1, C_1, $-A_1$, we have $T_2=\frac{1}{2}(B_1+A_1)$. We may remark that the sign of T_2 is of no importance in any application we are contemplating of these formulæ.

20. *Solution to a First Approximation.*—The complete solution of the preceding equations cannot be generally obtained. For their practical application they must be solved by approximation, when the approximate solution may be sufficient. The most important case is one in which the problem can be completely solved in consequence of its simplification arising from the particular conditions assumed. The case is that in which we suppose no forces to act at any point parallel to one of the coordinate axes, as that of z. In such case $C=0$. Also we assume the absence of any couple tending to twist a proposed element about the axis of x, or that of y, *i. e.* $D=0$, and $E=0$. Hence the equations ($c.$) (art. 16) become

$$\left.\begin{aligned}(A-p)\cos\alpha+F\cos\beta&=0,\\ F\cos\alpha+(B-p)\cos\beta&=0,\\ (\quad -p)\cos\gamma&=0;\end{aligned}\right\} \quad \ldots \quad (c'.)$$

and the cubic for the determination of p becomes

$$+\{(A-p)(B-p)F^2\}p=0.$$

This last equation gives for the value of p,

$$p=\tfrac{1}{2}\{A+B\pm\sqrt{(A-B)^2+4F^2}\},$$
$$p=0.$$

Or putting $(A-B)^2+4F^2=M^2$,

$$\left.\begin{aligned}p_1&=\tfrac{1}{2}\{A+B+M\},\\ p_2&=\tfrac{1}{2}\{A+B-M\},\\ p_3&=0.\end{aligned}\right\} \quad \ldots \quad (d'.)$$

For the values p_1 and p_2 of p, the third of the above equations ($c'.$) gives $\cos\gamma=0$; and the equation

$$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$$

gives

$$\cos\beta=\sin\alpha;$$

and eliminating p and β from the two first of equations ($c'.$), we obtain

$$\tan 2\alpha=\frac{2F}{A-B}. \quad \ldots \quad (e'.)$$

This corresponds to two values of α differing by 90°. These two values of α, and $\gamma=90°$ (since $\cos\gamma=0$), determine the two positions of the axes of the principal tensions p_1 and p_2. They both lie in the plane of xy.

Taking the third value of $p=p_3=0$, the equations (c'.) are also satisfied by

$$\cos\alpha=0, \quad \cos\beta=0;$$

by which the equation

$$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$$

is reduced to

$$\cos\gamma=\pm 1.$$

These values of α, β, and γ correspond to the axis of z, showing that axis along which the pressure $=0$ to be the third axis of principal tension.

21. In determining the magnitude and direction of T_2, the greatest tangential action, we must bear in mind the remarks in art. 19. The quantities denoted in the preceding formulæ by p_1, p_2, p_3 are the same as those denoted in art. 18 by A_1, B_1, C_1. The former are here retained for greater distinctness. If $A+B$ be $>M$, p_2 will be positive, and the order of the principal tensions will be p_1, p_2, p_3. The axis of y will then be the mean axis, and the direction of the maximum tangential action, T_2, will bisect the angle between the other principal axes of x and z. Also we shall have

$$\left.\begin{aligned} T_2&=\tfrac{1}{2}(p_1-p_3)\\ &=\tfrac{1}{2}\{A+B+M\}.\end{aligned}\right\} \quad \ldots\ldots\ldots \quad (f'.)$$

If, on the contrary, $A+B$ be $<M$, p_2 will be negative, and the order of the principal tensions will be p_1, p_3, p_2. The direction of T_2 will be perpendicular to the axis of z, bisecting the angle between the principal axes of x and y. Also we shall have

$$\left.\begin{aligned} T_2&=\tfrac{1}{2}(p_1-p_2)\\ &=M.\end{aligned}\right\} \quad \ldots\ldots\ldots \quad (g'.)$$

These two cases hold when

$$M<\text{or}>A+B$$

respectively, or

$$\left.\begin{aligned} (A-B)^2+4F^2&<\text{or}>(A+B)^2,\\ F^2&<\text{or}>AB;\end{aligned}\right\} \quad \ldots\ldots\ldots \quad (h'.)$$

or the second case may hold when A and B are both tensions, or both pressures, provided one of them be small; and it must necessarily hold when one is a pressure and the other a tension.

22. *Solution of the General Equations to a Second Approximation.*—In proceeding to second approximation I include C and E, but regard their magnitudes as small. These magnitudes must be expressed by the ratios which they bear to some standard force. The greatest value which the tangential force F can attain in any glacier must be limited by the *tangential cohesive power* of glacial ice; for if F exceeded this latter force, dislocation, by a tangential sliding of one element past another, would instantly ensue. Let this tangential cohesion be measured by F_1; then, when it is said that C and E are small,

it is meant that the ratios $\frac{C}{F_1}$ and $\frac{E}{F_1}$ are small ratios. This is equivalent to the assuming that the force on any element parallel to the axis of z is small, and that the couple with reference to an axis parallel to that of y, is also small. The condition $D=0$ signifies, as in the first approximation, the absence of a couple on every element, with reference to an axis parallel to that of x.

Hence putting $D=0$ in our general cubic, we have

$$(A-p)(B-p)(C-p)-\mathrm{I}\ (B-p)-F^2(C-p)=0,$$

or

$$\{(A-p)(B-p)-F^2\}(C-p)=E^2(B-p).$$

Putting $C=0$ and $E=0$, we have, as before, for the three first approximate values of p,

$$\left.\begin{array}{l} p_1=\frac{1}{2}\{A+B+\sqrt{(A-B)^2+4F^2}\}, \\ p_2=\frac{1}{2}\{A+B-\sqrt{(A-B)^2+4F^2}\}, \\ p_3=C. \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (d''.)$$

To proceed to a second approximation, put

$$p=p_1+\varpi_1$$

in the cubic, and we obtain

$$\{(\overline{A-p_1}-\varpi_1)(\overline{B-p_1}-\varpi_1)-F^2\}(\overline{C-p_1}-\varpi_1)=(B-p_1-\varpi_1)E^2,$$

or

$$\{(A-p_1)(B-p_1)-F^2-(A-p_1+B-p_1)\varpi_1+\varpi_1^2\}(C-p_1-\varpi_1)=(B-p_1-\varpi_1)E^2.$$

The first approximation gives

$$(A-p_1)(B-p_1)-F^2=0;$$

and the preceding equation shows that ϖ_1 must be of the order E^2. We may therefore neglect terms in $\varpi_1 E^2$, and ϖ_1^2, and we then obtain

$$-(A+B-2p_1)(C-p_1)\varpi_1=(B-p_1)E^2,$$

and

$$\varpi_1=-\frac{p_1-B}{A+B-2p_1}\,\frac{E^2}{p_1-C};$$

or, since C is small,

$$\varpi_1=\frac{p_1-B}{2p_1-(A+B)}\cdot\frac{E^2}{p_1^2}\left(1+\frac{C}{p_1}\right)p_1.$$

ϖ_2 and ϖ_3 may be found in the same manner.

23. Again, equations (*c.*), art. 16, become, if $D=0$,

$$\left.\begin{array}{r} (A-p)\cos\alpha+F\cos\beta+E\cos\gamma=0, \\ F\cos\alpha+(B-p)\cos\beta=0, \\ E\cos\alpha+(C-p)\cos\gamma=0, \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (c''.)$$

The two last equations give

$$\cos\alpha=-\frac{C-p}{E}\cos\gamma,$$

$$\cos\beta=\frac{F}{B-p}\,\frac{C-p}{E}\cos\gamma.$$

If we were to substitute these expressions in the first equation, we should obtain the cubic in p used in the immediately preceding articles for the approximate determination of its value. Substituting the values of cos α and cos β in the equation

$$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1,$$

we obtain

$$\left\{\left(\frac{p-C}{E}\right)^2+\left(\frac{F}{p-B}\right)^2\left(\frac{p-C}{E}\right)^2+1\right\}\cos^2\gamma=1,$$

or

$$\left\{(p-C)^2\left(1+\frac{F}{(p-B)^2}\right)+E^2\right\}\cos^2\gamma=E^2.$$

The retention of E^2 in the coefficient of $\cos^2\gamma$ would only produce a term in the expression for $\cos^2\gamma$ of the order E^4, and may therefore be neglected. Also, since p only differs from its first approximate value, p_1, by a small quantity of the order E^2, we may, for the reason just assigned, write p_1 for p in the last equation. Thus we have

$$\cos\gamma=\pm\frac{E}{(p_1-C)\sqrt{1+\left(\frac{F}{p_1-B}\right)^2}},$$

and by substitution,

$$\cos\beta=\frac{F}{p_1-B}\cdot\frac{1}{\sqrt{1+\left(\frac{F}{p_1-B}\right)^2}},$$

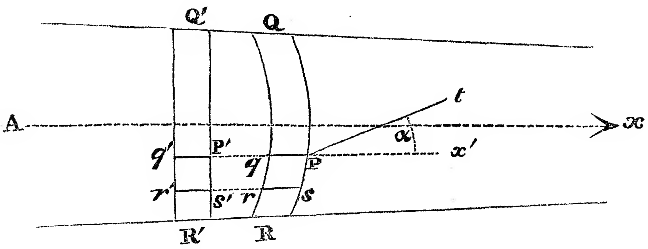
$$\cos\alpha=\frac{1}{\sqrt{1+\left(\frac{F}{p_1-B}\right)^2}}.$$

These equations show that γ is changed from a right angle to one whose difference from a right angle is of the order E, and therefore small; while cos α and cos β are changed by small quantities of the same order as the difference between p and p_1, *i. e.* of the order E^2.

24. *Nature of the Forces* A, B, C, D, E, *and* F *in the ordinary cases of Glaciers.*—In the preceding investigations we have considered the forces A, B, C, D, E and F as acting on any element of a solid body. In the case of a glacier, this body assumes a specific form, and it becomes necessary to explain what forces are represented by the above symbols in this particular and restricted case. The phenomena with which we shall be here concerned, have been observed almost entirely in those regions of glaciers in which most large ones, like those of the Alps, become much elongated in consequence of the narrowness of the valleys down which they descend. The sides of these valleys frequently approximate to parallelism with each other. The primary general characteristics of the motion of glaciers of this kind are (1) the motion is unaccelerated, (2) the axial portions move with a greater velocity than the marginal portions, and (3) the superficial portions move somewhat faster than the lower portions of the mass. These points are clearly established by observation, independently of any particular theory. In the following articles of this section their truth is assumed.

25. Let fig. 2 represent the section of a glacier by a plane parallel to its surface, and fig. 3 a vertical section through A x the axis of the glacier. Let A x be taken as the

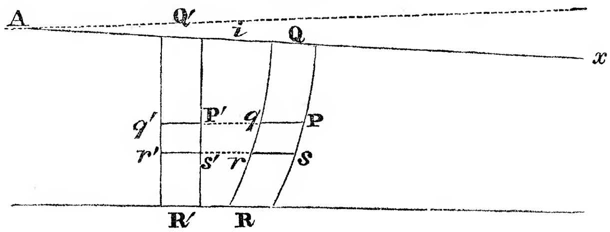
Fig. 2.



Section parallel to the surface.

axis of x, and the surface of the glacier (nearly horizontal) as the plane of xy, a transverse line perpendicular to A x being the axis of y, and a perpendicular to the surface of the glacier the axis of z. Let Q′ R′ (fig. 2) represent a section, by a plane parallel to that of xy, of an element of the mass lying between two planes perpendicular to the axis of x; Q′ R′, by the more rapid motion of the axial parts of the mass, will be brought into the position Q R. Also if Q′ R′ (fig. 3) represent a section of the same element by a vertical plane parallel to that of xz, Q′ R′ will be brought into the position Q R by the more rapid motion of the upper surface of the mass. Also the small elementary parallelopiped whose section parallel to (xy) is represented by P′ $q' r' s'$ in fig. 2, will be brought into the position P $q\, r\, s$, while the section P′$q'r's'$ (fig. 3) of the same element made by a

Fig. 3.



Vertical section.

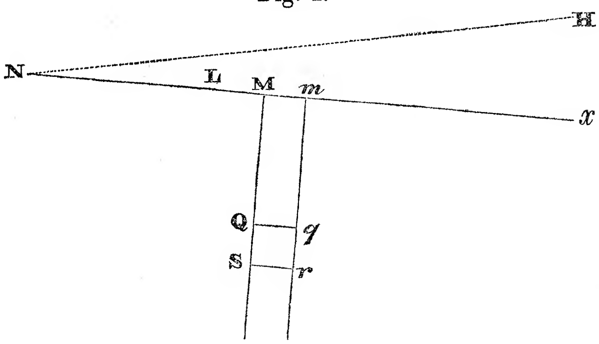
plane parallel to (xz) will be brought to the position P $q\, r\, s$. Thus we see that there will be an *angular distortion* of P $q\, r\, s$ about an axis parallel to z, as represented in fig. 2, and a similar distortion about an axis parallel to y, as represented in fig. 3. These angular distortions are respectively due to the couples whose intensities are represented by F and E (art. 15). Also it is easily seen that there will be no twisting or angular distortion of P $q\, r\, s$ about an axis parallel to that of x, and that consequently D will $=0$ in all such glaciers as we are now considering. A becomes a *longitudinal* tension, and B a *transversal* one. C will be the pressure on the element parallel to z, and due to the superincumbent weight, and the inclination of the surface of the glacier to the horizon.

26. The intensity of A and B will depend much on the form of the glacial valley. If the sides be parallel, A may be a tension or pressure according to the local variations in

the form of the bed of the valley; B will be small. If the sides be convergent in descending the valley, A will almost necessarily be a pressure, on account of the resistance which the sides will oppose, by their convergency, to the onward progress of the glacier. B will become a great pressure, greater than A, to which it will bear somewhat the same relation as the pressure on the side of a wedge bears to that applied to its back. If the valley be divergent, and if when it becomes so, its inclination is very much diminished (as at the lower end of the Rhone glacier) A may become an enormous pressure, while B may become a tension on account of the lateral expansion which will be given to the mass by the great pressure *à tergo*. C will be best considered in conjunction with E. D, as above stated, will always $=0$. F will have different values for different points in the same vertical transverse section of the glacier. It will manifestly be greatest in the marginal portions, where the angular distortion represented in fig. 2 is greatest; in the central portions, the motion of the glacier will produce very little of this angular distortion about an axis parallel to that of z, and F will be proportionally small.

27. The forces C and E are more dependent on each other than A, B, and F. I proceed to investigate expressions for them. For this purpose let fig. 4 represent a vertical section of the glacier parallel to (xz) and not too remote from its axis; then will F, as above stated, be very small, and may be neglected. Also $D=0$, and B acts perpendicular to the plane (xz), and will therefore not affect the relations between the forces acting parallel to that plane.

Fig. 4.



Let x, y, z be the coordinates of the point Q. We shall have $NM=x$, the distance of the plane N M Q from that of $xz=y$, and $MQ=z$. We might obtain the results required by considering the conditions of equilibrium of the element $\delta x\delta y\delta z$, represented by Q q r S; but it will be more convenient to take an element represented by M Q q m, of which the volume will be $z\delta x\delta y$. I suppose here the existence of a longitudinal pressure parallel to the axis of x, and represented in our general formulæ by A. If we draw a plane through M Q perpendicular to the axis of x, A may represent the intensity of the longitudinal pressure at any point on that plane, referred to a unit of surface. For different points in this plane x will be constant, and the pressure may vary generally with y and z; but it will answer our immediate purpose, and much simplify our problem, if we suppose A constant for every point in M Q, or independent of y and z. It will then vary only in passing from any plane M Q to a consecutive and parallel plane, *i. e.* A will be a function x alone. Hence we shall have

Pressure on one of the sides of the element M q perpendicular to the plane of $yz=A\,.\,z\delta y$,

Pressure on the opposite side $=-\left(A+\frac{dA}{dx}\delta x\right)z\delta y$.

The algebraical sum of these pressures, estimated in the positive direction of x,

$$=-\frac{d\mathrm{A}}{dx}z\delta x\delta y.$$

Again, if W be the weight of a unit of volume of ice,

$$\text{Weight of the element } \mathrm{M}q=\mathrm{W}.z\delta x\delta y,$$

and its resolved part parallel to the axis of x

$$=\mathrm{W}z\delta x\delta y\sin\iota.$$

Also we shall have the tangential force on the base Qq,

$$=-\mathrm{E}\delta x\delta y,$$

parallel to the axis of x.

Hence we must have for a condition of equilibrium of the element Mq

$$-\frac{d\mathrm{A}}{dx}z+\mathrm{W}z\sin\iota-\mathrm{E}=0,$$

and

$$\mathrm{E}=\left(-\frac{d\mathrm{A}}{dx}+\mathrm{W}\sin\iota\right)z.$$

It would seem probable that A will generally vary slowly with y; it may vary more rapidly with z, especially in cases where it becomes large, as at the bottom of an ice-fall. In such case we shall have

$$\mathrm{A}=\int_0^z\frac{d\mathrm{A}}{d\zeta}d\zeta,$$

where ζ is taken parallel to z. In the position just mentioned (the foot of an ice-fall) the variation with regard to x may possibly be rapid, and therefore $\frac{d\mathrm{A}}{dx}$ very considerable. Under the ordinary conditions of a glacier, away from any rapid fall, the variations of A must generally be slow, and the values of $\frac{d\mathrm{A}}{dx}$ therefore comparatively small.

C is manifestly due to the resolved part of the pressure of Mq on the surface Qq referred to a unit of surface. The normal pressure on Qq=W.$z\delta x\delta y\cos\iota$. Whence

$$\mathrm{C}=\mathrm{W}z\cos\iota.$$

This value of C shows that it must always be small when z is so. Such will also be the case with E, unless $\frac{d\mathrm{A}}{dx}$ be very large, which is probably true only at the foot of an ice-fall. Generally, then, we see that C and E will be small for all those depths which lie within the sphere of our observation; and that for all such depths the first approximate solutions of our general equations are sufficiently accurate. The second approximate solutions give the results for greater depths, and indicate the nature of the results for those still greater depths at which C and E might be too large to render the results of the second approximation applicable with sufficient exactness.

We may now distinctly understand the interpretation of the first approximate solutions of our general equations, as applied to the case of an actual glacier. In those

solutions it has been assumed that C, D, and E are so small as to be neglected. D always $=0$, and it follows from the preceding expressions for C and E that, generally, they are relatively small for all those more superficial portions of a glacier to which our observations can extend. Hence, for the same portions, our first solutions will be approximately true and practically applicable.

SECTION IV.—*On the manner in which Dislocations in the Mass of a Glacier, or in its Structure, may be produced; and on the resulting Phenomena.*

28. It will be recollected that the internal pressures and tensions which have been investigated in the preceding pages, are those which would exist in a continuous solid mass acted on by certain external forces, previous to the dislocation which must result from such forces if the intensity of the internal tensions should be sufficient to overcome the cohesion of the mass, or the pressures to overcome its resisting-power. We may, however, carry our geometrical and mechanical analysis of the problem somewhat further, and consider *how* the dislocation will take place when the forces are sufficient to produce it. It has already been remarked (art. 2) that there are three ways in which this may occur. In the first place, the cohesion may give way to the greatest normal tension, p_1; open fissures will then be formed. Again, when the maximum compression (p_2) becomes very great, it may be easily conceived how the primitive structure may break down, as it were, especially if the mass be of a crystalline structure like ice. The third kind of dislocation is that produced by the tangential action between two contiguous elements, which obviously tends to make one element slide past the other, and thus to produce what Principal FORBES has called a "differential motion." I shall consider successively these different kinds of dislocation, and the phenomena which respectively result from them, with reference, in the first place, to the more superficial portions of the glacier, in which our first approximate formulæ are applicable; and secondly, the possible formation of similar phenomena in the deeper parts of the glacial mass.

From the equations $(d'.)$, art. 20, we have

$$p_1=\tfrac{1}{2}\{A+B+\sqrt{(A-B)^2+4F^2}\},$$
$$p_2=\tfrac{1}{2}\{A+B-\sqrt{(A-B)^2+4F^2}\},$$
$$\tan 2\alpha=\frac{2F}{A-B}.$$

The last equation gives two values (α_1 and α_2) of α, which determine the directions of p_1 and p_2 with respect to the axis of the glacier. A, B, C, and F, in the application of the formulæ to an actual canal-shaped glacier, are such as described in arts. 25 and 26. We have also the result that the lines of maximum tangential action at any point bisect the right angles between the directions of maximum tension and maximum pressure.

Of the values α_1 and α_2 of α, one will be greater and the other less than 90°; and we must determine, in each problem, which gives the maximum and which the minimum

pressure p_1 and p_2. It may be convenient to consider F an absolute positive quantity; and we may suppose, for such a glacier as that represented in fig. 2, that $B=0$. In such case the preceding formula shows that one of the values (as α_1) of α must be small and positive when F is small, *i. e.* for any point P (fig. 2) near the axis $A x$. Now the maximum tension at P must be due to the tension A parallel to $A x$, and that produced by the angular distortion of the element $P q r s$. The magnitude and direction of this latter tension are obtained by putting $A=0$ and $B=0$ in the above formulæ. This gives the greatest tension $=F$, the least $=-F$, and $\tan 2\alpha=\infty$, or therefore $\alpha=45°$ or $135°$. From the inspection of the element, it is manifest that the first of these values of α corresponds to the greatest tension produced by the angular distortion. It is from this tension and A that p_1 in the actual case of a glacier before us, must result. Consequently p_1 must act in some such direction as $P t$ (fig. 2), where α in that figure is acute. The same result will hold if B be of finite magnitude, and algebraically less than A; and thus α_1 and α_2 are distinguished from each other in the case before us, and by similar reasoning may be distinguished in any other case.

29. *Formation of Transverse Crevasses.*—When the maximum normal tension is the force to which the cohesive power of a glacial mass first gives way, the result, as above observed, must be an open fissure, or crevasse, the direction of which must manifestly be perpendicular to that of the tension producing it. These crevasses approximate more or less to right angles with the glacial axis, and usually characterize canal-shaped valleys in which the sides are approximately parallel. In such cases B must be comparatively small; if the valley be slightly convergent, it will be a small pressure, and therefore negative. When these crevasses exist more abundantly, A will doubtless be a large tension, though not necessarily so, as we shall see, for the production of a crevasse.

(1) Taking the simplest case, let us first suppose $A=0$ and $B=0$. This may be very approximately true if the glacier descend without acceleration or retardation along a trough-like valley of uniform width and uniform inclination. We shall then have by the above equations, $p_1=F$, $p_3=-F$, and $\tan 2\alpha=\infty$. Hence the direction of maximum tension will make an angle $\alpha=45°$ with the axis of the glacier $(A x)$ (fig. 2) towards which it will converge in descending. If, therefore, a fissure be formed at all, which can only be in the marginal regions where F is considerable, it must be in a direction perpendicular to $P t$, and making an angle of 45° with the axis of the glacier. We have also for the maximum tangential action (art. 21),

$$T_2=\tfrac{1}{2}(p_1-p_2)=F,$$

equal, in this case, to p_1, and making an angle of 45° with the directions of p_1 and p_2. It will therefore be parallel to the axes of x and y. Hence the maximum normal and tangential forces make equal efforts, in this case, to dislocate the mass. Let the tangential cohesive power of the mass be F_1; the greatest value which p_1 can assume will then be also F_1; and if the normal cohesive power (P_1) be less than F_1, the tension p_1 may assume a value (F) between P_1 and F_1, by which a crevasse will be formed in the position above mentioned. Transverse crevasses may therefore be formed without any of

that direct longitudinal tension which might, at first sight, appear necessary to produce them.

(2) If there be a considerable longitudinal tension, but no appreciable transverse pressure, we have

$$p_1=\tfrac{1}{2}\{A+\sqrt{A^2+4F^2}\},$$
$$p_2=\tfrac{1}{2}\{A-\sqrt{A^2+4F^2}\},$$
$$\tan 2\alpha=\frac{F}{A}.$$

Hence p_1, the maximum tension, will be increased, and therefore, also, the tendency to form a crevasse. Likewise the greatest value of α will be less than 45°, and the direction of the crevasse, as we might expect, will be more nearly perpendicular to the axis of the glacier.

(3) Many glacial valleys become narrower as we descend them, and consequently the mass of the glacier may enter each part of the valley as a wedge, and may frequently become more or less compressed. In such case B will be negative, and may become very large. We shall then have

$$p_1=\tfrac{1}{2}\{A-B+\sqrt{(A+B)^2+4F^2}\},$$
$$p_2=\tfrac{1}{2}\{A-B-\sqrt{(A+B)^2+4F^2}\},$$
$$\tan 2\alpha=\frac{F}{A+B}.$$

In this instance, as well as in the preceding one, p_1 will necessarily be a tension, and greatest (A and B being constant) where F is greatest, *i. e.* in the marginal portions of the glacier. For the like reason, α will also be greatest in those portions. It will vanish at the axis, where F vanishes. Hence, if the forces be sufficient to overcome the cohesion of the mass, a curvilinear crevasse may be formed extending across the glacier and meeting its axis at a right angle. This, however, is rarely the case, the transverse crevasses being formed, in general, in the lateral portions only of canal-shaped glaciers, where they will approximate more or less to straight lines. They are most likely to be formed where A is a considerable tension, which is less likely to be the case in converging valleys.

It is important to observe that in all cases in which the expression for tan 2α is positive, *i. e.* where B is algebraically less than A, α must lie between 0 and 45°, and consequently *the inclination of a curved crevasse to a transverse line perpendicular to the axis must likewise always lie between the same limits.* This rule is applicable, according to this theory, wherever transverse crevasses are likely to be formed*.

* An open curvilinear fissure in its *progressive* formation would be in some degree influenced by other causes than the maximum tension at each point through which it might pass. Moreover, the cohesive power has been above supposed to be the same in every direction from any proposed point. There *may*, on the contrary, be planes of *less cohesion*, in which case if the cohesion along any such plane bear a smaller ratio to the internal tension perpendicular to it, than the cohesion perpendicular to the maximum tension bears to p_1, the crevasse may be formed along the plane of least cohesion. I know no reason, however, to suppose that these causes are sufficient to modify in any essential degree the law enunciated in the text.

30. *Formation of Longitudinal Crevasses.*—Transverse crevasses, as above stated, are almost invariably formed in valleys the sides of which are approximately parallel; longitudinal crevasses are as generally formed in valleys in which the sides are divergent, and usually perhaps when they become rather suddenly so. They are found almost exclusively at the lower ends of glaciers. The Rhone glacier affords the best-known example of crevasses of this kind, but M. AGASSIZ refers to a number of other cases, the greater part of which, however, appertain to small glaciers*. At the foot of the great ice-fall of the Rhone glacier, the valley expands largely, as is well known, and its inclination to the horizon becomes comparatively small. Thus the ice, accumulating at the bottom of the fall, exerts an enormous pressure *à tergo* on the ice immediately before it, and this pressure is propagated onwards in directions which radiate from the bottom of the fall. It is manifest that the force along each radiating line will be a great pressure. Again, if we conceive the whole mass divided into concentric rings perpendicular at each point to the above-mentioned radiating lines, the pressure along these lines will extend the ring, and produce in it a great tension at every point, perpendicular to the direction of the radial pressure above mentioned. Hence if we take any one of these radiating lines as the axis of x, the radial pressure at any point upon it will be denoted by $-$A, and the tension upon it in the direction perpendicular to the axis of x, will be B. The former will be a *principal pressure*, and the latter a *principal tension*; and if crevasses be formed at all, they must be in directions perpendicular to that of B; *i.e.* they must be radial, as they are always observed to be. The glacier of the Rhone is only a type, as regards longitudinal crevasses, of all other glaciers in which they exist.

31. An explanation of the phenomena of transverse crevasses, essentially the same as that above given, though founded on a more restricted mechanical and mathematical analysis of the general problem, was given by me some seventeen years ago. I am not aware that any doubt was entertained as to its validity, but I am also not aware of any glacialist having recognized it previously to Dr. TYNDALL. M. AGASSIZ has given an explanation† both of the transverse and longitudinal crevasses, involving apparently the notion of the oblique tension which, I have proved, must necessarily exist in a determinate direction. The mechanical reasoning employed, however, is too vague to constitute a mechanical explanation of the phenomena. Moreover his work was published in 1847, three or four years after my memoir containing the preceding explanation was printed in the Transactions of the Cambridge Philosophical Society. Principal FORBES, also, speaks of a *drag* from the sides towards the centre of a glacier‡, but with the view apparently, not of explaining the formation of transverse crevasses, but of the veins in cases of the veined structure. His "lines of greatest strain" must therefore, I conceive, mean the same lines mechanically, not as my lines of greatest *normal* tension, to which the crevasses are unquestionably due, but my lines of greatest *tangential* action. If he supposed these two directions to be identical, it was a manifest error; for

* Système Glaciaire, p. 324. † Ibid. p. 320 *et seq.*

‡ Occasional Papers, p. 57; also last chapter of his 'Travels.'

I have proved that they must in all cases differ by an angle of 45°. I have thought it right to say thus much to vindicate my claim to having been the first to give any explanation, founded on true mechanical principles, of the phenomena in question.

32. That the theoretical law above enunciated with respect to the directions of transverse crevasses is the actual law, is determined, I conceive, beyond all doubt. It was, in fact, so distinctly recognized by M. AGASSIZ and others*, before careful observations had been made on glacial motion, that it was regarded as a proof that the sides of a glacier moved faster than its central portion. But the best and, I believe, the only accurate recorded evidence on the subject is that afforded by the admirable map published by M. AGASSIZ, of the glacier of the Aar. There the crevasses, in a particular locality, are laid down with geometrical accuracy, and the eye recognizes at once the law in question, notwithstanding the minor deviations which must necessarily result in such a case from local and irregular causes†. Principal FORBES, however, does not seem to recognize this law; for he observes‡ that he agrees with some preceding observers in believing crevasses to be mere *accidents* of glacial motion. I have, however, demonstrated, as I had done before the publication of the paper in which this opinion is expressed, that though a glacier should be affected by no irregularities of surface along its margins or bed tending to dislocate it, the internal tensions arising from the more rapid motion of its central portions would always tend to produce oblique transverse marginal fissures. Local and irregular causes may frequently counterbalance this tendency; but the cause itself which produces these fissures is no more *accidental* than the law of the motion in which it originates. It might as well be asserted that the longitudinal or radiating crevasses of the lower part of the glacier of the Rhone were merely accidental phenomena,—an assertion which few glacialists, I imagine, would venture to maintain.

33. The positions of the crevasses as above determined, will be those in which they are originally formed. It is manifest that transverse crevasses will change their angular positions with reference to the axis of the glacier, by the more rapid motion of its axial portion. They will thus become more approximately perpendicular to the axis. Moreover, as they deviate from their original positions, less force will be exerted to keep them open, and to counteract any independent or local causes tending to close them. They do thus finally disappear, and generally at points not remote from those where they were originally formed, and where local conditions probably aided their formation. A closing up of the crevasses after a certain time, presents no difficulty; but the entire

* Système Glaciaire, p. 305.

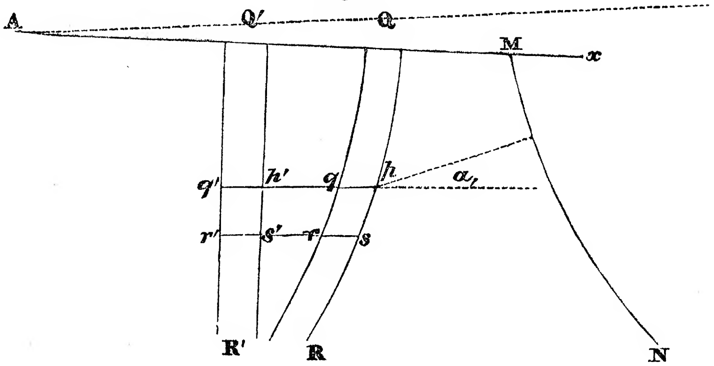
† I cannot here omit the expression of my conviction of the inadequate justice which has been rendered in this country to the 'Système Glaciaire' of M. AGASSIZ. It professes to be, as it is, principally descriptive, and contains more accurate details on many points than any other work professes to offer. The map which accompanies it, founded on the very careful trigonometrical survey of M. WILD, is full of beautifully delineated details. We possess no other glacial document of the kind at all comparable to it. I shall have other occasions to refer to it.

‡ Occasional Papers, p. 151.

disappearance of all traces of them, and the resumption of complete crystalline continuity in those parts of the mass in which these crevasses had previously existed, remained a difficulty of which no physical explanation had been given till the experiments of Dr. TYNDALL afforded so happy a solution of it by proving that the renewed continuity was attributable to regelation resulting from the renewed contact of portions of the ice which had been previously separated.

34. *Formation of Crevasses in the deeper portions of a Glacier.*—The formation of crevasses in this position must ever perhaps remain a matter of speculation; for we can have little hope of making their existence a matter of observation. The general direction of maximum tension at any point of a glacier at a great depth, cannot be exactly determined, because the general equations (*c.*), art. 16, cannot, for this case, be generally solved. It will be easy, however, to determine this direction in terms of the internal forces for any point in the vertical plane through the axis of the glacier, and, consequently, the section made by that plane with the surface to which the direction of the maximum force is a normal—the surface along which the crevasse would be formed. For this purpose let fig. 5 represent the vertical section through the axis of the glacier.

Fig. 5.



The undistorted element Q′ R′ will be brought by the more rapid motion of the surface into the position Q R, as described in fig. 3. The internal forces acting on the small element $p\,q\,r\,s$ will be A, B, C, and E, the latter producing the couple whose axis is parallel to that of y. It is manifest, however, from the conditions of symmetry, that the direction of the maximum tension must lie in the vertical plane through the axis of the glacier, and will not be affected by B, which, as in other cases of transverse fissures, must be supposed either a pressure or a comparatively small tension. Hence, putting $-$C for B and E for F in our formulæ (e'.), art. 20, we have

$$\tan 2\alpha = \frac{2E}{A+C}.$$

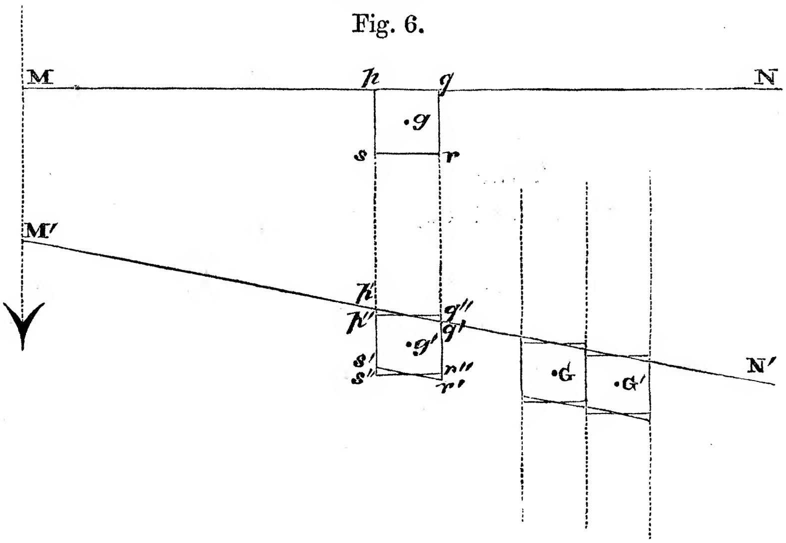
The angle α is indicated in fig. 5 (see art. 28 and fig. 2). Near the surface, E and therefore α, vanish, and the crevasse, if formed, will be vertical. At lower points of the mass α will increase with E, and the vertical section of the crevasse will resemble the curve M N in the figure.

The maximum tension in the marginal portions of the glacier will be increased by F; and it is in those portions, near the lower as well as near the upper surface of the mass, that crevasses will be most likely to be formed. The forces tending to form them may probably be much the same in both cases; but the tendency of the incipient fissures to open into wide crevasses must almost necessarily be much counteracted near the lower surface by the action of the bottom of the valley on that surface. The sides of a fissure will not there be able to separate from each other with the same facility as at the free surface of the mass. In fact any incipient fissure formed within the glacier and not extending to its external surface, will have much less of this facility of subsequently opening, than those which are formed nearer to the outer surface and directly communicate with it. I therefore conceive it to be very improbable that crevasses exist in the deeper portions of a glacier, equal in number and magnitude to those which are seen in its more superficial portions.

35. *Effects of the Crushing of the Mass.*—It may frequently happen that the maximum pressure (p_2) at any proposed point of a glacial mass shall be much greater than the maximum tension p_1, or the maximum tangential force T_2 (art. 18) at that point; and the dislocation may be produced by the crushing power of p_2. The element may thus be broken into a greater or smaller number of fragments, and the constraint of the mass removed. Its onward motion will then be continued, and its continuity restored by regelation, till a repetition of the process becomes necessary to overcome a new state of constraint. It is in this manner that the property of regelation enables us so beautifully to account for the molecular mobility of a glacial mass, and the consequent freedom of its central to move more rapidly than its marginal portions, without any reference whatever to the property of viscosity or plasticity.

36. *Dislocation produced by the Tangential Force.*—The third way in which the mass may be dislocated is, that the tangential cohesion may yield to the tangential force on any proposed element, as explained in article 28. For the greater simplicity of explanation, I shall suppose the sides of the valley parallel, and A=0, and B=0. Also we take C=0 and E=0 as in our first approximate solutions. Let $p\,q\,r\,s$ (fig. 6) be the

Fig. 6.



section of a small elementary rectangular parallelopiped (δx, δy, δz) made by the plane of the paper (that of xy); and let $p\,s$ be in the direction ($p\,p'$) of the motion of p; it will be parallel to x, and the transverse line M$p\,q$N, perpendicular to $p\,p'$, will be parallel to y. Now the physical line M N will, in a short time, come into the position M′ N′, by the assumed motion of the glacier, and the greater velocity of its axial portion. The element $p\,q\,r\,s$ will then come into the position $p'\,q'\,r'\,s'$, and will be angularly distorted. It will be acted on by the tangential force (F), forming two equal and opposite couples, with a common axis parallel to z (art. 29, (1)), and tending to destroy the continuity on every side of the element, by overcoming the tangential cohesion. The force F will be different for different angular positions of the element, and will always be greatest when the sides of the element bisect the two right angles between p_1 and p_2, the greatest tension and the greatest pressure (art. 18). In the present case the directions of p_1 and p_2 will each be inclined at an angle of 45° to the axes of x and y (art. 29, (1)); and therefore the intensity of the forces (F) of the two couples will be greatest when they act respectively parallel to x and y, i. e. when the element is in the angular position represented in the figure.

Now if tangential dislocation take place, it must necessarily do so in those directions in which the tendency of the forces F to produce it is greatest, or, in the case before us, in directions parallel and perpendicular to the axis of the glacier, the tendencies being the same in both those directions. Let us suppose, then, a complete tangential rupture to take place, and simultaneously, on each side of the element $p\,q\,r\,s$, as well as in the surrounding elements. Each element, represented by $p'\,q'\,r'\,s'$, will then regain its original rectangular form by its *elasticity*, but so moving that its centre of gravity (g') shall remain at rest, since the elastic force of restitution acts entirely within the element. Thus $p\,q\,r\,s$ will return to its original rectangular form $p''q''r''s''$; and if we take two consecutive elements whose centres of gravity are G and G′, they will be brought into the relative positions represented in the figure, in which G′ is slightly in advance of G, as much, in fact, as is necessitated by the more rapid motion of the central parts of the glacier. After the rupture, if we suppose the continuity to be instantaneously restored (as it will be by regelation according to our theory), the glacier will again be brought as a continuous mass into a position of no constraint by its general motion. By a repetition of this process, the continuity of the mass will be constantly destroyed and as constantly restored; and we thus see the *modus operandi* by which, taking any two elements situated like G and G′, the one nearest the axis of the glacier gets gradually in advance of the other, as it must do, in accordance with the general law of the glacier's motion.

37. But, it may be asked, if the dislocation takes place on the two sides of the element whose directions are transverse to the glacier, simultaneously with the rupture along the sides whose directions are longitudinal, why is it that the subsequent relative or differential motion of two contiguous elements, such as G and G′, should not be transversal as well as longitudinal? The reason is obvious. The motion which instan-

taneously takes place after the tangential dislocation is a motion of rotation *about* G, the centre of gravity of the element, by which that point is not affected; whereas the motion with which we are here concerned, as that which alone can possibly produce any finite differential motion, is the motion of the centre of gravity (G) *itself*, arising from the general onward progress of the whole mass. Any real differential motion between two particles must, in all cases (whatever, in fact, may be the directions of tangential dislocation), take place in the actual direction of the motion of the particles. Thus, if the directions of tangential dislocation be not parallel and perpendicular to the axis of the glacier, as in the above case, the true differential motion of two particles must still take place in the actual direction of their motion.

38. In the above case we have assumed the rupture to be complete and simultaneous on every side of the element, and also the absence of friction between contiguous elements. If it be otherwise, as it doubtless will be, the same reasoning will be applicable; but the relief of the constraint of the mass, or of any element, at each dislocation will be only partial, and the consequent differential motion will be somewhat less. In such case the facility with which the central portions of the mass move faster than the other parts will be diminished.

39. I am not here supposing that this tangential dislocation is the most probable mode by which the constraint of the glacial mass is commonly relieved under great pressure. The crushing of its elementary portions (art. 35) would appear, perhaps, a more likely *modus operandi*; but possibly both these processes may contribute to the actual dislocations (without finite fissures) by which the constraint of the mass is instantaneously relieved. If the dislocations were entirely tangential, it is easily seen that two contiguous elements, like G and G′ (fig. 6), would ultimately be separated, while each should preserve its physical identity. Consequently, identical particles forming at one time the continuous transverse linear element, M N, might subsequently be converted into an elongated loop which should be discontinuous in the marginal parts of the glacier, where the differential motion would be greatest, and continuous in the central portions, where that motion would be least. If, on the contrary, each element should be *crushed* in the dislocation, it would manifestly never regain its primitive form, but would become compressed or extended *as if* it were plastic or viscous (art. 4, &c.). In such case the original transverse element M N would be converted into a continuous loop. But, at all events, whether the real *modus operandi* consist of only one of the above processes, or of a combination of both, it is of the first importance for a just appreciation of the principle of regelation, that we should see distinctly how the continuity of the glacial mass is broken and again restored by it, in contradistinction to the effect of real viscosity, which would prevent that continuity from being broken at all. In the latter case also there would be no particular structure, such as the crystalline structure, to be destroyed, and no necessity for the power of regelation to restore it. It is in the difference between the *modus operandi* when the mass is viscous, and that when

the mass is crystalline and brittle, that we see best, perhaps, the distinction between the Viscous and Regelation Theories.

SECTION V.—*On the Veined Structure of Glacial Ice.*

40. Two different theories, as is well known, have been proposed to account for the curious structure frequently observed in glacial ice, and termed the laminar, veined, or ribbon structure. The preceding investigations bear intimately on these theories. One of them, that of Principal FORBES, asserts the structure to be due to the sliding of one thin lamina of ice past another, or to their differential motion—a process by which he supposes extensive portions of the mass to be divided into slices, usually nearly vertical, and not exceeding, in many cases, the fraction of an inch in thickness. His first idea was that these parallel discontinuities were, after their formation, filled with infiltrated water, which, by being frozen, formed the veins of blue ice. He afterwards appears to have abandoned this notion; but what physical process he supposed to be substituted for infiltration I have not been able distinctly to ascertain. This, however, is not material as regards the examination which I propose to give this theory in the sequel. The other theory above alluded to is that first proposed by Dr. TYNDALL, an essential point in which is that the laminæ of ice which characterize this structure must be perpendicular at each point to the direction of greatest pressure there. So far alone my researches are related to this theory, and so far only shall we be here concerned with it. I shall not discuss the manner in which this pressure is supposed to produce the lamination in question, a point on which somewhat different opinions have been propounded. It is the mechanical part only of the theory that I shall discuss. It has been called the *pressure theory* of the veined structure, while that advocated by Principal FORBES has been termed the *differential theory* of the structure. The lines in which the laminæ of blue or white ice meet the surface of the glacier will generally be curved lines, which I shall designate as *lines* or *curves of structure*.

This structure, as well as the lines depending upon it, have received different designations according to the portion of the glacier where it is found, or the directions of the lines of structure. Thus, when found in the lateral portions of the glacier, Dr. TYNDALL has termed it the *marginal structure*. It is frequently found in canal-shaped glaciers. The lines of structure in such cases are usually inclined at small angles to the sides of the glacial valley. When the lines stretch more directly and entirely across the valley, the structure is said to be *transverse*, and is more especially developed at points below an ice-fall and not remote from it. In the axial part of a glacier the lines of structure are frequently longitudinal, or parallel to the axis. The structure is then called *longitudinal*. It is generally best exhibited at the confluence of two large glaciers, as those of the Finsteraar and Lauteraar. It may be convenient to preserve these designations, though, as will be seen, they are the results, according to the pressure theory, of the same general cause, modified by the local conditions under which it acts. I take again

the first approximate solutions of our equations, as applicable to all usually accessible depths.

41. *Pressure Theory of the Laminated Structure.*—We have generally,

$$p_1=\tfrac{1}{2}\{A+B+\sqrt{(A-B)^2+4F^2}\},$$

$$p_2=\tfrac{1}{2}\{A+B-\sqrt{(A-B)^2+4F^2}\},$$

$$\tan 2\alpha=\frac{2F}{A-B}.$$

In the formation of crevasses, we have been principally concerned with p_1 and its direction, we shall now be more especially concerned with p_2, when a pressure, and the corresponding value of α; but it will be convenient to bear in mind that, according to the fundamental principle of this theory, the curve of structure through any point will always coincide with the direction of p_1, the maximum tension or minimum pressure at that point, p_2 being always the maximum pressure there. Also, since the direction of p_2 is horizontal (neglecting C and E as in our first approximation), each laminar surface must be vertical, and will therefore be completely determined by the line of structure corresponding to it.

42. *Formation of the Marginal Structure.*—I have stated that this is frequently found in canal-shaped glaciers. Suppose the glacial valley to be more or less convergent, so that the transverse force B may become a pressure; and suppose its magnitude to be greater than that of A, as it probably will be generally in such a valley. We may also suppose A to be a pressure, though this is not essential for the production of the laminar structure in the case before us. The preceding formulæ now become

$$p_1=\tfrac{1}{2}\{-A-B+\sqrt{(B-A)^2+4F^2}\},$$

$$p_2=\tfrac{1}{2}\{-A-B-\sqrt{(B-A)^2+4F^2}\},$$

$$\tan 2\alpha=\frac{2F}{B-A}.$$

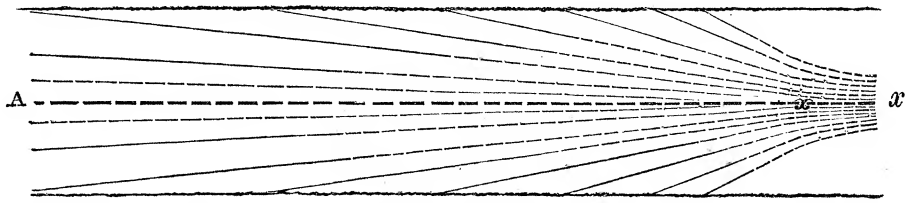
p_2 will be the maximum pressure, and the structural curve at any proposed point will be perpendicular to the direction of p_2; it will therefore coincide with that of p_1, which will be determined by the above expression for $\tan 2\alpha$. Let α_1 be the value of α which gives the direction of p_1, or that of the curve of structure at any proposed point. Then will α_1 lie between 0 and 45°. At points near the axis of the glacier, the *twisting* tendency, and therefore F also, will be very small, and may be neglected. We shall then have

$$p_2=-B.$$

Let us first suppose B too small to produce the veined structure. It will not, in such case, exist at all in the central portion of the glacier. At any point more remote from the axis, p_2 will be increased by the increase of F, and may become sufficient to produce the structure. α_1 will be greatest when F is so, *i. e.* at the sides of the glacier, supposing A and B to remain constant. Should B be much larger than F, as we should antici-

pate in the cases we are considering, α_1 will be much less than 45°, the greatest value to which it can attain when the transverse pressure B predominates over the longitudinal pressure A. The lines of marginal structure will be as represented in fig. 7, by the continuous portions of the oblique lines.

Fig. 7.



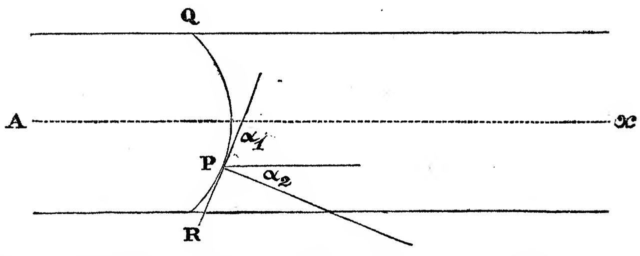
43. *Formation of the Longitudinal Structure.*—If B, the transverse pressure, be very great, p_2 may be sufficient to develope the veined structure in the central as well as the marginal parts of the glacier. The curves of structure will then be continued towards the axis of the glacier, to which they will constantly converge as an asymptote, as represented by the discontinuous lines in fig. 7. If B be very large, the lines of structure, when examined only in a limited area, will be sensibly parallel to each other and to the sides of the glacier.

44. *Formation of the Transverse Structure.*—Let us now suppose the longitudinal pressure A to be very great, as it must be, for instance, at the bottom of an ice-fall. We may also suppose it much greater than B. We shall then have

$$\tan 2\alpha = -\frac{2F}{A-B},$$

and consequently 2α will either be a small negative angle, or positive and nearly equal to 180°; and therefore α will either be small and negative, or positive and nearly equal 90°. The former must, from the nature of the case, correspond to the direction of maximum pressure, and must therefore be α_2, and the other α_1, as in fig. 8. α_2 will $=0°$ at

Fig. 8.



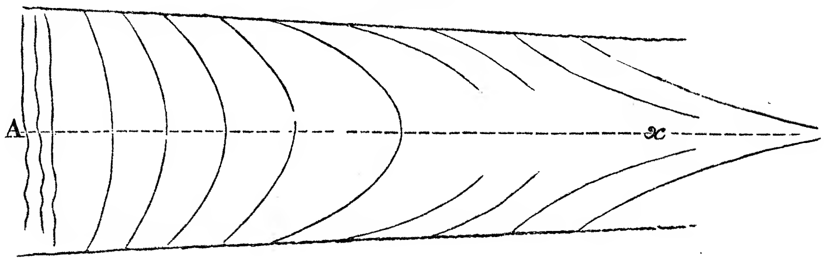
the axis, and will be greatest at Q and R. If A be sufficiently great, the curve of structure will be continued across the glacier, constituting the transverse structure.

45. There are two cases connected with the formation of the transverse structure at the bottom of an ice-fall which ought to be noticed. It may happen that the valley below the fall shall be an elongated slowly *converging* valley, or, as occurs perhaps more rarely, a rapidly diverging one like that already described at the extremity of the Rhone

glacier. In the first case, especially if the inclination of the valley should gradually increase and become considerable, the transverse pressure, B, may be much increased, and the longitudinal pressure, A, much diminished as we descend the valley, till it becomes less than B. In such case the curves of structure (so far as they depend alone on the action of the external forces) would become more and more elongated, as represented approximately in fig. 9. If B should become greater than A,

$$\tan 2\alpha = \frac{2F}{B-A},$$

Fig. 9.



and the case becomes the same as that previously considered (art. 43), in which the formation of the longitudinal structure is explained. So long, however, as A remains a pressure sufficient to produce the laminar structure at right angles to the axis, the loops will be completed; if A become too small to produce the structure, the curves will not be continued across the axis, and the structure will exist only as a marginal structure; and if B still increase (by the glacial valley becoming very narrow), so as to produce the longitudinal structure along the axis, the structure will become longitudinal, as represented by the two lower lines of structure in fig. 9.

46. In the second case above mentioned, of a rapidly divergent valley of small inclination, the mass will be urged onwards by a great radial force, as above explained (art. 30), and the curves of structure will be perpendicular to the crevasses, and consequently approximately concentric about the foot of the ice-fall, where this structure commences.

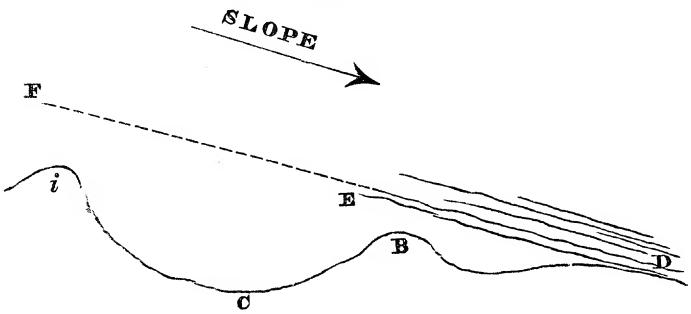
47. *Application of the preceding Theoretical Results.*—It must be recollected that the phenomena of glacial structure as they actually exist at any moment, are not merely the results of the instantaneous action of the mechanical and physical causes to which their original formation may have been due, but to those causes together with the effect produced by the motion of the glacier. In the formulæ of this section I have entirely neglected the influence of transmission by this motion, on the forms of the curves of structure, and have spoken of them as though they were dependent only on the forces originally producing them, and unaffected by the unequal motions of the central and marginal portions of the glacier. This, however, cannot be the case, unless we suppose the structure to be modified at every instant in exclusive obedience to the physical and mechanical causes in which it originates—a supposition having a great apparent improbability. The motion of a glacier being known, it is merely a geometrical problem to

point out accurately the nature of the modification which would be produced in a given curve or surface of structure by transmission alone; the amount of that modification can only be known by observation. I shall investigate this subject in a subsequent section. At present, in the application of the preceding formulæ to particular cases, my conclusions will be restricted to the hypothesis of the structure being due alone to the instantaneous action of the forces tending to produce it in each particular locality.

48. The *marginal structure* is found more or less developed in most canal-shaped glaciers. It has been best observed, perhaps, on the glacier of the Aar and its tributaries, and on the glaciers at Chamouni. It sometimes coexists with marginal crevasses. In such cases the curves of structure ought to be, according to the pressure theory, perpendicular to the crevasses. Moreover, the coexistence of the structure and crevasses implies a longitudinal tension and a transverse pressure; in which case the direction of greatest tension will make an angle of much less than 45° with the axis or sides of the glacier, and such, therefore, will also be the case with the marginal curves of structure. This seems accordant with the best observations; for though the forms and positions of these curves have not yet been observed with all the care they require, there is little doubt as to their meeting the sides of the glacier at finite angles. The law of perpendicularity between the structural curves and the crevasses was first observed, I believe, by Principal FORBES on the glacier of the Aar; and in speaking of the Talèfre glacier* he remarks, "The crevasses as they present themselves are convex towards the origin of the glacier, and here, as in other cases, perpendicular to the veined structure." The same impression appears to have been produced on other observers.

49. It is manifest that the marginal structure may be modified by any local conditions which affect the internal pressure of the glacier. Principal FORBES describes what he regards as an extraordinary case, and one to which he appeals as affording a crucial test in favour of his own views†. It is furnished by the glacier of La Brenva, and is represented by fig. 10. The line i C B D represents the side of the glacial valley,

Fig. 10.



in which projections at B and D appear to form a somewhat sudden obstacle to the onward motion of the glacier in the direction F D. Now it is certain that the maxi-

* Occasional Papers, p. 192.

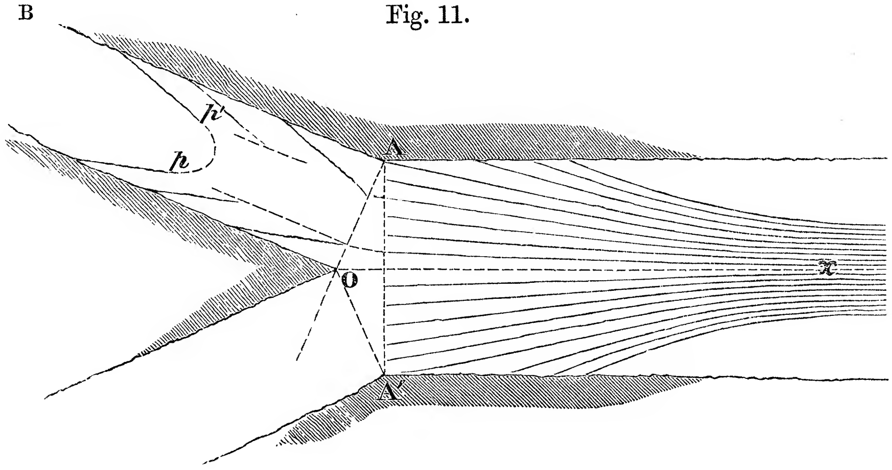
† Ibid. p. 56.

mum pressure between B and D would be very great, and that its direction would be nearly perpendicular to that of the motion of the glacier. Consequently the direction of the veins, according to the pressure theory, ought to coincide approximately with the direction of the motion, as represented by the lines between B and D. The example, as I understand it, presents no difficulty in the pressure theory*.

50. The *transverse structure* is well developed near the bottoms of the ice-falls of the upper part of the Glacier du Géant, and that by which the ice is precipitated from the glacier of Talèfre to that of Léchaud; but the most striking exhibition of it is afforded by the lower end of the Rhone glacier, which has already been mentioned (art. 30) as affording an excellent example of longitudinal, or, more properly, radiating crevasses. The curves of structure, according to the law above stated (art. 48), are perpendicular to the crevasses, and therefore coincident with the surfaces of greatest pressure, as they ought to be according to the pressure theory.

51. The *longitudinal structure* is perhaps best developed below the junction of the Finsteraar and Lauteraar glaciers, where they form the glacier of the Aar†. The marginal structure is well exhibited along the flanks of both the tributaries, and is continued along those of the combined glacier resulting from their union. It is from the point of junction O, fig. 11, and on each side of the axis O x, that the longitudinal

Fig. 11.



structure is so finely developed. The tributary B A represents the Lauteraar glacier. The oblique lines on either flank represent the marginal structure, which is supposed to extend only to a certain distance from the sides. The whole width of the glacier of the Lower Aar is much less than the sum of the widths of its two principal tributaries, so that the transverse pressure (especially near the axis), for some distance below the

* I may remark that I see nothing distinctive in the observations made by Principal Forbes on La Brenva to determine the relative velocities of points at different distances from the side of the valley. Exactly similar results would be obtained on any glacier in which the velocity of the central should be considerably greater than that of the marginal portions.

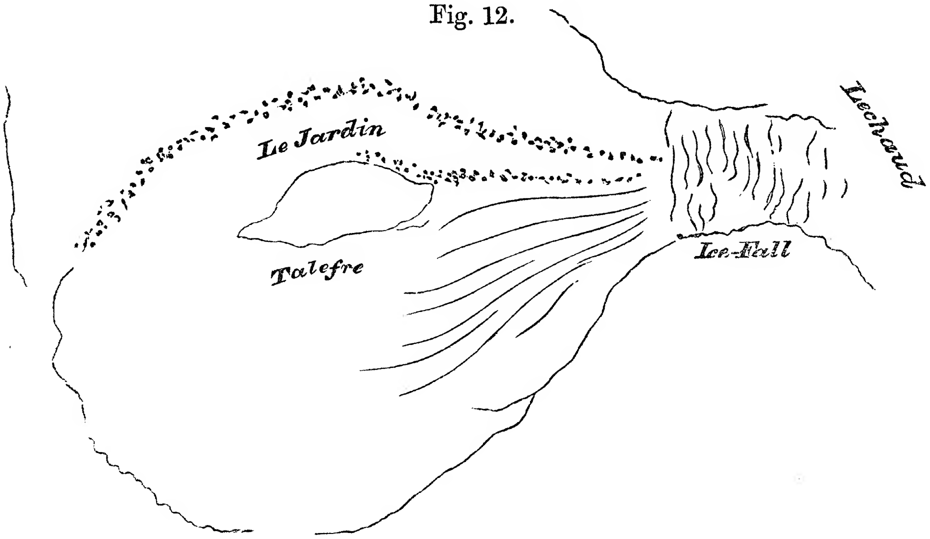
† Tyndall's 'Glaciers of the Alps,' p. 387.

junction, must manifestly be enormous. About some such section as O A this pressure will begin to be felt, and will be in full operation about A A′. Now considering the direct effects which would be produced by the forces acting on the united glacier below the section A A′ (independently of the effects of transmission), that portion will present precisely the case of longitudinal structure considered in art. 43. The transverse pressure, as above remarked, will be enormous, especially along the axis, to which it will there be perpendicular, and the general structure will be such as is represented in fig. 7, and repeated in fig. 11. The maximum pressure at any point in the axial part of the glacier will be increased by the mutual action of the two tributaries after their confluence, required to divert each from its original direction; and in the marginal parts of the great glacier the maximum pressure will be increased by the comparatively large value of F. In the region intermediate to the marginal and central portions, the pressure will probably be less than in either of those portions, but in the figure it is supposed to be sufficient to superinduce a longitudinal structure. In this case, however, the angle α at any point P would not increase regularly, and the curves in the part of the glacier now referred to might present some degree of inflexion.

According to Dr. TYNDALL, the longitudinal character of the veined structure is strongly developed, as a general rule, under all considerable central moraines; and as such moraines always originate in the confluence of two glaciers, and present conditions similar to those of the Aar, the explanation in all such cases will be precisely similar to that above given, assuming the structure to be due to the instantaneous action of the forces acting on the mass, and not to transmission.

52. In the glacier of Talèfre the ice-stream is divided by the Jardin into two separate currents, as roughly represented in fig. 12, which thus form, in fact, two separate tribu-

Fig. 12.



taries to the united glacier which proceeds, below the Jardin, to precipitate itself over the ice-fall of Talèfre. Principal FORBES appears to have examined the veined structure carefully, and describes it as represented in the figure by the fine lines converging to the

fall. The dotted lines are moraines, and indicate the course of the ice-current. It is manifest that the mass, in approaching the fall, must suffer immense transverse compression, which accounts for the longitudinal structure in that part of the glacier.

53. *Formation of the Veined Structure in the deeper portions of a Glacier.*—We cannot determine completely the direction of the greatest pressure at any proposed point at a great depth in a glacier, for the reason above assigned (art. 34). That direction would give us the normal to the surface of greatest pressure, and therefore, also, to the surface of structure through that point, whence the differential equation to the surface would be known. In our practical inability to follow this method, we may proceed, as in art. 34, to determine the section of the required surface made by a vertical plane through the axis of the glacier. In a glacier of considerable width, the sections of this surface made by planes parallel to the one just mentioned, will be very similar to the axial section; because the velocities will be very nearly the same for all points in a transverse line on the surface of the glacier, within considerable distances of the axis*; and the same therefore must also hold for points at greater depths. The most important case is that which occurs at the base of an ice-fall in which A becomes very large and a pressure. Also B, the transverse force, may be considered much smaller than A, and a pressure. Our general formulæ will become

$$p_1=-\tfrac{1}{2}\{A+C-\sqrt{(A-C)^2+4E^2}\},$$

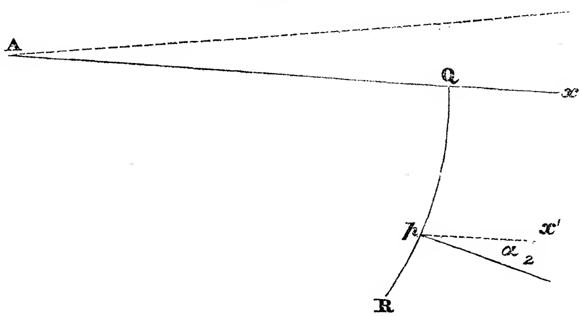
$$p_2=-\tfrac{1}{2}\{A+C+\sqrt{(A-C)^2+4E^2}\},$$

$$p_3=-B;$$

and we shall have, also,

$$\cos\beta=0,\ \tan 2\alpha=-\frac{2E}{A-C},$$

Fig. 13.



the plane of α being now that of xz. Now B being supposed small compared with A, p_2 will be the maximum pressure and p_1 the minimum pressure, or algebraically the maximum tension. The above equation gives two values of α. Now 2α must be negative and less than 90°, or positive and between 90° and 180°; and therefore α must be negative and less than 45°, or positive and between 45° and 90°. The negative value will evidently here correspond to p_2 the greatest pressure. A may possibly be much the same at different depths, in our present case, and E (art. 34) will be zero at the surface, and will increase with the depth. C also increases with the depth, as does, therefore, *cæteris paribus*, the negative angle α_2. It must always, however, be less than 45°, so long as C is less than A, whatever may be the depth; and near the surface it will vanish.

* M. AGASSIZ has best exemplified this in his admirable map and diagrams of the glacier of the Aar already referred to. He there delineates (Atlas, pl. 4) the forms assumed by an originally straight transverse physical line on the glacier near the junction of its two great tributaries in three successive years. The motion of different points on the central portion of the mass do not differ much throughout a breadth of nearly one-half of the glacier.

The axial section of the surface of structure may be represented by Q R in fig. 13, varying in its inclination to the nearly vertical axis of z from zero to less than 45°. A similar conclusion will hold, as above stated, for all the central portion of the glacier.

The structure here considered is manifestly a transverse structure, and is probably only formed where there is a sudden change of inclination in the bed of the glacier, like that at the foot of an ice-fall. The intensity of the longitudinal pressure to which it is there due, may possibly diminish rapidly in receding from the fall; and so far it would follow that, where the structure is continued to any considerable distance from the locality in which it was formed, it must be due in a great degree to transmission. But this is a question which I reserve for further discussion.

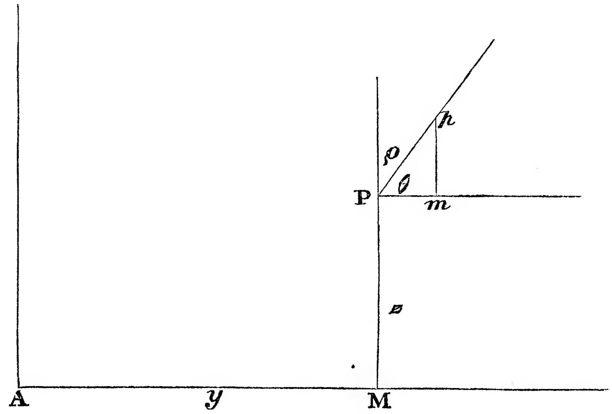
54. *Differential Theory of the Veined Structure.*—The idea on which this theory is founded has already been stated at the beginning of this section (art. 40); and the modes in which dislocation may take place so as to admit of the more rapid motion of the central portion of a glacier, have been explained. It has also been pointed out (art. 36) that the real differential or relative motion of two contiguous particles of the mass must necessarily be in the common direction in which the particles are constrained to move, by virtue of the external conditions to which the whole glacier is subjected. This, in fact, belongs to the definition of "differential motion" as I understand the term; nor can I conceive how any mechanical effects, such as dislocation or bruising, can be attributed to a "differential motion" in any arbitrary or conventional sense, rather than to the real relative motion which I understand to be meant by the expression. If so, it appears to me impossible to accept this theory of the veined structure without a far more explicit explanation than any which has yet been given of it. Assuming, then, what I conceive to be the only intelligible and determinate meaning of the above expression, I shall proceed to investigate certain geometrical results which flow from it. Since the investigation is entirely geometrical, we may suppose the surface of the glacier and its longitudinal axis to be horizontal. I shall also suppose the line of motion of every particle to be parallel to that axis, and the velocity to be invariable along each such line, but different for different lines, (1) because the velocity of the central is supposed to be greater than that of the marginal portions of the mass, and (2) because the velocity of its upper is greater than that of its lower surface.

55. Let P (fig. 14) be any particle of the glacier, and let the plane of the paper represent a transverse plane perpendicular to the common direction of all the lines of motion of the component particles of the mass. Also let p be any other particle likewise in the plane of the paper, its distance from P being very small and equal to ϱ; i. e. p must lie in the circumference of a circle in the plane of the paper, whose centre is P, and whose radius ϱ is extremely small. Our first object will be to determine the relative velocity, or differential motion of P and p, and the positions of p in the small circle when that relative velocity is a maximum, and when it is zero.

For this purpose let the horizontal plane of the base of the glacier be made the plane of xy, and the vertical plane parallel to the longitudinal axis of the glacier, and along its

flank on the left, the plane of xz. The plane of yz will be a vertical transverse plane; let it be represented by the plane of the paper, to which, therefore, the line of motion of every particle will, by hypothesis, be perpendicular. Let P (fig. 14) represent a

Fig. 14.



material point in the plane of the paper, or that of yz. (We take it in this particular plane because we shall not be immediately concerned with the coordinate x.) Let $AM=y$, and $MP=z$. Also, take another point p supposed to be very near P, and let its coordinates be $y+\eta$ and $z+\zeta$, η and ζ being referred to P as origin. Also let V be the velocity of P perpendicular to the plane of the paper, and $V+v$ that of p; V will be some function of y and z; and v a function of η and ζ. As we have chosen the coordinate planes, V will increase with y and z, because the centre of the mass moves faster than its sides, and the upper moves faster than the lower surface; and for the like reasons, v will increase with η and ζ. Now since η and ζ are very small, we may consider any increase of v, due to an increase of η, as $=\mu\eta$, and an increase due to that of ζ, as $=\mu'\zeta$, where μ and μ' are functions of y and z, but independent of η and ζ. They express the rate at which V increases as we pass from P to contiguous points in the plane of the paper, and in directions parallel respectively to the axes of y and z. We shall then have

$$v=\mu\eta+\mu'\zeta;$$

or if

$$\eta=\varrho\cos\theta,$$
$$\zeta=\varrho\sin\theta,$$
$$v=\varrho(\mu\cos\theta+\mu'\sin\theta).$$

From what has been above stated, this quantity must be a maximum with respect to θ, considering μ, μ', and ϱ* as constants. This gives

$$-\mu\sin\theta+\mu'\cos\theta=0.$$

* The tendency of the tangential differential motion of two particles very near together, to bruise or dislocate the mass, will manifestly depend on the contortion or twisting produced by this relative motion, and therefore on the angular change of position of the line joining the two points. Thus, in fact, $\frac{v}{\varrho}$ is the quantity to be really made a maximum. This is equivalent to considering ϱ constant in differentiating as in the text.

Let θ_1 and θ_2 be the two values of θ which satisfy this equation; then

$$\tan\theta_1 = \tan\theta_2 = \frac{\mu'}{\mu};$$

and

$$\theta_2 = \tan^{-1}\frac{\mu'}{\mu} + 180^\circ.$$

From the above expression for v, θ_1 gives v a positive value, and θ_2 the same numerical value with an opposite sign, the one being the algebraical maximum value, the other the minimum one.

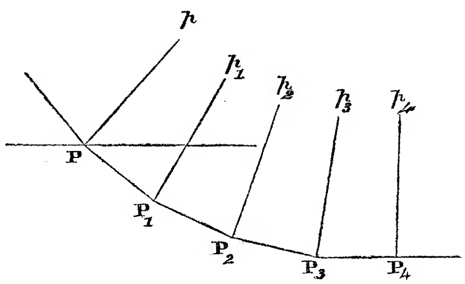
Moreover, we see from the above value of v, that $v=0$ when

$$\tan\theta = -\frac{\mu}{\mu'} = -\cot\theta_1,$$

which shows that the angular distance between the directions in which v is respectively a maximum and zero, is equal to a right angle.

56. To interpret these formulæ, draw $\mathrm{P}p$ (fig. 15) making the angle $\tan^{-1}\frac{\mu'}{\mu}$ with the axis of y. The relative velocity for p and P will be greater than for any other point at the same distance from P. Draw $\mathrm{P\,P_1}$ perpendicular to $\mathrm{P}p$, taking $\mathrm{P\,P_1}$ very small; then since $v=0$ in the direction at right angles to $\mathrm{P}p$ (by the last equation), the velocity of $\mathrm{P_1}$ will equal V, that of P. At $\mathrm{P_1}$ (since it is nearer the central axis of the glacier than P) the *rate* of increase of V, as depending on y, will be less than at P, and therefore μ will also be less (art. 55); and for a like reason* (since $\mathrm{P_1}$ is nearer the lower surface than P) μ' will be increased. For both these reasons $\theta\left(=\tan^{-1}\frac{\mu'}{\mu}\right)$ will be greater at $\mathrm{P_1}$ than at P. Draw $\mathrm{P_1}\,p_1$ accordingly. Again, draw $\mathrm{P_1\,P_2}$ perpendicular to $\mathrm{P_1}p_1$, making $\mathrm{P_1\,P_2}$ very small. The velocity of $\mathrm{P_2}$ will still $=$V, and the relative velocity of p_2 and $\mathrm{P_2}$ will be a maximum in the same sense as before. We may proceed in the same manner with any number of points. Now, when we pass to the limit, by taking $\mathrm{P\,P_1}$, $\mathrm{P_1\,P_2}$, &c. indefinitely small, the locus of $\mathrm{P\,P_1\,P_2}$, &c. will become a continuous curve, possessing the dynamical property, in the case of a glacier, that each of its particles moves with the same velocity perpendicular to the plane of the paper, and they have consequently no tendency to separate from each other.

Fig. 15.



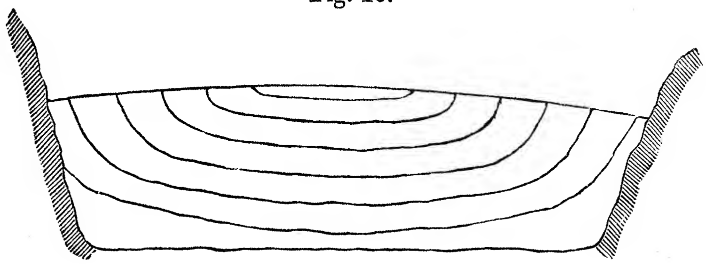
Again, taking ϱ indefinitely small, we shall have another continuous curve $p\,p_1\,p_2$, &c. contiguous to the former, and so related to it that the relative velocity of two particles situated respectively on these curves, and on a common normal to them, will be a

* The increase in the *rate* at which V increases as we descend from one longitudinal line of motion to another, cannot be made a matter of observation, but the analogy with the variation of V in passing from the axis of the glacier to its sides (in which case we know that the *rate* of variation increases) would seem fully to justify the assumption. The conclusion of the text, however, is not dependent upon it.

maximum, and will consequently have a maximum tendency to separate from each other by their differential motion. I have spoken here of physical *points* as perhaps conveying more distinctly the idea of the intervening distance denoted by ϱ. I might have equally spoken of indefinitely small elements of determinate forms, considering ϱ as the distance between their centres of gravity. These are only different kinds of phraseology by means of which we reason on the ultimate elements of which a continuous mass must be constituted.

In the above investigation the velocity of each particle has been assumed to be the same at every point along its line of motion. Consequently whatever holds for the motions with which these particles pass through one transverse plane, will hold with respect to any other such plane. Consequently there will be two contiguous cylindrical surfaces generated by lines parallel to those of the motion of the glacier, and having for their directors the two contiguous curves above described on the plane of yz; and the motion of the particles in these cylindrical surfaces will have the same dynamical characters as those above enunciated for the particles in the two guiding curves. There will be an indefinite number of such surfaces following the same law. A section of the mass, made by a transverse vertical plane parallel to that of yz, will be represented by fig. 16. Each surface will be perpendicular to this transverse plane.

Fig. 16.



This system of surfaces would necessarily result from the differential motion in the sense in which I regard it; nor can I conceive how the veined structure can originate in that motion, in the way in which the author of the theory appears to consider it to originate, unless the veins should coincide with the surfaces here investigated. In such case every line of structure on the surface of the glacier, in the typical case above considered, would coincide with a line of motion, and would therefore be parallel to the axis and sides of the glacier. In the central parts of the glacier, where the variation of V for different points situated on a transverse horizontal line is usually very slow, μ will be very small, and therefore $\theta\left(=\tan^{-1}\frac{\mu'}{\mu}\right)$ will nearly equal 90°. Consequently the structural surfaces will be very nearly horizontal. In the marginal portions of the glacier these surfaces will meet the superficial surface of the mass at angles

$$=90^\circ-\theta=90^\circ-\tan^{-1}\frac{\mu'}{\mu},$$

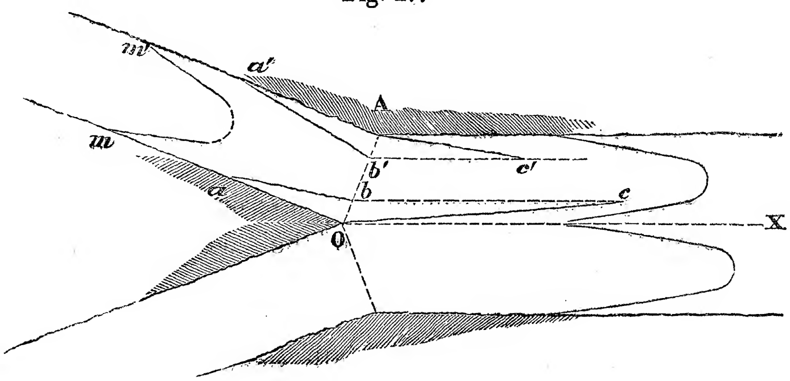
which will be nearly $=90^\circ$, since μ' will be usually small near the upper surface, and μ

will be comparatively large near the sides of the glacier. The superficial curves of structure would be all straight lines parallel to the axis of the glacier.

In actual glaciers, the forms of the curves of structure on the outer surface would be modified by the fact of the unequal thinning of the glacial mass in consequence of the more rapid thawing towards its lower extremity, so that the existing external surface is no longer parallel to the longitudinal lines of motion. Consequently the outer surface would intersect the internal structural surfaces obliquely, and the superficial curves of structure would be changed from straight parallel lines to very elongated loops, but would never approximate to straight transverse lines, as they always do at the foot of an ice-fall. We may also observe that there could be no longitudinal development of veins along the axis of the glacier, as the immediate result of differential motion. Such veins could only exist by transmission, as will be explained in the next section, where I shall revert to the forms of the surfaces of greatest differential motion which have been here investigated.

57. *Modification of the Veined Structure arising from the Motion of the Glacier.*—I now proceed, according to my intention as above expressed (art. 47), to examine the modifications produced by the general motion of a glacier, on the forms and positions of the lines and surfaces of structure as originally produced by the immediate action of the causes to which they may be due. Such modifications as can be observed on the surface of a glacier, are principally due to the more rapid motion of its central portion; the interior surfaces of structure must also be modified by the more rapid motion of the upper surface of the mass. In an ordinary canal-shaped glacier, it is manifest that the marginal curves of structure will thus be brought into more approximate parallelism with the sides, if originally inclined to them. This modification will be small. A greater one will be produced in the transverse structure, such as is usually found at the foot of an ice-fall. There the superficial curves of structure run almost directly across the glacier, by the motion of which they will be transformed into elongated curves, at distances from the fall sufficiently great, the elongation increasing with that distance. In a glacier, like the Aar, formed by the junction of two great tributaries, the modification of structure below their confluence will be more complicated. Let fig. 17 represent the glacier as

Fig. 17.



before, and let $a\,b$, $a'b'$ be two lines of marginal structure, representing the two systems of lines to which they respectively belong. A particle in the tributary glacier will move parallel to the side of the glacier till it arrives, for instance, at b, a point near the section O A, and will afterwards move parallel to the axis O X. When the particle at a arrives at O, suppose the particle which was simultaneously at b to have arrived at c, $b\,c$ being parallel to O X; then will O c be the position in the united glacier of the physical line of structure which before occupied the position $a\,b$. But the velocity with which a particle will move from b to c in the central part of the united glacier, will be considerably greater than that with which a particle will move from a to O along the side of the tributary. Consequently the distance $b\,c$ will be considerably greater than $a\,b$, and O c will make a proportionately smaller angle with O X than $a\,b$ makes with the side a O of the tributary. Hence, if the lines of marginal structure approximate to parallelism to the sides in the tributary, the same lines, forming those of the longitudinal structure in the central portion of the united glacier, will approximate still nearer to parallelism with each other and with O X. Subsequently this approximation to parallelism will decrease, since a particle will move rather faster along O X than along $b\,c$; but assuming always that the divergency is small in the marginal structure of the tributary, a simple calculation shows that it will not become considerable in the united glacier except at distances from the junction equal to many multiples of the whole breadth of the glacier*. Such, at least, would be the case with the glacier of the Aar. If, on the contrary, the lines of marginal structure in the tributaries should make a considerable angle with the sides of the glacier, they would also make a considerable, though smaller angle with the axis of the united glacier, and their inclination to that axis would afterwards increase more rapidly.

Taking the opposite flank of the glacier, suppose two particles on the same line of structure to be simultaneously at a' and b'; and reasoning as before, suppose b' to have moved parallel to the side of the united glacier to c' while a' moves to A. Then will A c' be the resulting position of the *same* line of structure as previously occupied the position $a'\,b'$. In this case, however, $b'\,c'$ will be nearly equal to a' A, since the velocities along the margins of the combined glacier and the tributaries will probably be much the same. Hence the directions of the marginal lines of structure with reference to the sides of the valleys, will be less changed than those of the longitudinal structure with reference to the axis O X.

I have above supposed the absence of any transverse lines of structure on the central portion of the tributary glaciers. If, however, they exist so as to complete the loops of the curves $(m\,m')$ (fig. 17), they will of course also exist, by the transmission we are here assuming, in the combined glacier, in the forms of similar loops, as represented in the figure.

58. If we compare the forms of the marginal structural curves in a canal-shaped glacier (represented in fig. 7, art. 42), resulting from the immediate action of the forces

* This calculation is founded on data supplied by M. AGASSIZ in his 'Système Glaciaire,' p. 451.

producing them, it is evident that it will generally be difficult or impossible to recognize by observation the modification which may have been produced in them by transmission. The case of structure represented in fig. 9 (art. 45), if the real structure in a glacier were sufficiently developed and carefully observed, would afford a test as to whether the forms were original, or had been modified by transmission; for, in the latter case, the loops would never change by elongation into an absolutely longitudinal structure as represented in the figure referred to. But this test is far, at present, from being a practical one. Again, in the glacier of the Aar, if the structure were perfectly developed, it would afford us the required test, as is easily seen by comparing fig. 17 and its completed loop in the main glacier, as transmitted from the tributary, with fig. 11 and the lines of longitudinal structure in the main glacier. If the glacier exhibited either of these structures complete, it would testify at once to the point in question. I am not aware, however, that the structure, to whatever cause it may be due, is sufficiently developed, or has been observed with sufficient care, to afford any positive testimony on the subject. The glacier of the Rhone, also, fails to afford us a practical test on this point; for the curves of structure may there be accounted for either by supposing them instantaneously formed or transmitted from the fall where they originate.

The supposition that the structural surfaces, at points of a glacier remote from the place at which they commence, are merely the effects of transmission, involves the conclusion that the veined structure can only be originally formed under enormous pressure (according to the pressure theory), like that at the bottom of an ice-fall, and that, when once formed, it is difficult to obliterate. In this case it would seem that the transverse curves of structure formed at the bottom of a fall ought to be distinctly preserved at a distance from it, as elongated loops extending completely across the glacier. I am not aware how far they are so, either in the case in which they proceed from the Talèfre fall, for instance, on the Mer de Glace, or from that of the Géant. The glacier of La Brenva above cited (art. 49) seems to afford an example of the instantaneous generation of the veined structure under a pressure which may not be comparable with that immediately below the junction of the Aar tributaries; and this same example at La Brenva seems also to afford an instance of a comparatively sudden obliteration of the structure where the immediate cause of it ceases to act. We may also remark that the marked and increased development of the longitudinal lamination along the central moraines proceeding from the junction of two considerable glaciers, appears to indicate the local efficiency and instantaneous effect of the causes of the structure. But particular cases of this kind require to be examined with greater accuracy of detail before we can derive from them any reliable inferences. It would seem most probable that the structure in any particular locality may frequently be due partly to the instantaneous action of physical causes, and partly to transmission. The question well deserves the consideration of those glacialists who may interest themselves about this curious structure of glacial ice, and the physical and mechanical causes to which it may be due.

I may make another remark on this doubtful question. The *dirt-bands* of the Mer de Glace, if Dr. TYNDALL'S views respecting them should be proved to be correct, may afford the means, supposing their forms and positions sufficiently determinate, of deciding how far the loops of structure in the middle and lower parts of the glacier may be merely the original transverse curves at its upper end, modified by transmission, or may be attributable to the great transverse pressure to which that glacier must be subjected at certain points of its course. If the dirt-bands are entirely formed at the upper part of the glacier and are there coincident with the curves of transverse structure, and if they also remain coincident with them at distant points of the glacier, then, since the dirt-bands must necessarily be transmitted forms, the curves must almost necessarily be so likewise. Principal FORBES'S theory of the dirt-bands would not lead to the same conclusion.

59. *The Pressure Theory and the Differential Motion Theory of the Veined Structure compared.*—The pressure theory of the veined structure, so far as it asserts the perpendicularity of such surfaces to the directions of maximum pressure, appears to be in perfect accordance with observed facts and mechanical deductions, whether the structure be marginal, transverse, or longitudinal. The transversal directions and approximate verticality of the structural surfaces at the bottoms of ice-falls, and the general existence of the structure wherever the glacier must, from the conditions under which it is placed, be subject to great pressure, are also perfectly consistent with this theory. We may also remark that the evidence of facts in favour of it is in a great measure independent of the degree of efficiency which may ultimately be attributed to the transmission of the structural forms. I am not, indeed, aware of any leading observed facts of the veined structure inconsistent with this theory.

With respect to the Differential Theory, the whole of the mechanical reasoning on which it is based is professedly popular, vague, and undemonstrative, and, I believe, as erroneous as such reasoning must almost necessarily be in cases as intricate as those which glaciers present to us, unless it be guided by an accurate conception and a careful analysis of problems which admit of more or less accurate solution, and are typical of those presented to us in nature. In the marginal structure, this theory could never agree with any very sensible deviation in the directions of the superficial curves of structure from parallelism with the sides of a canal-shaped glacier. The attempt to correct this defect by means of what has been called the Ripple Theory, will not now, I imagine, be maintained by any glacialist. The constant existence of the veined structure under great pressure, and its comparative absence where such pressure cannot exist, are left by this theory without satisfactory explanation. It fails altogether to assign any direct cause for the great development of the longitudinal structure along the central moraine of compound glaciers like the Aar; for along the axes of such glaciers there cannot possibly be any differential motion which could produce it. The only structure which could there exist must be a transmitted structure.

But the most conclusive objection to the differential theory is to be found, as I believe,

in its total inability to account for the highly developed structural surfaces, their nearly vertical positions, and approximate perpendicularity to the axis of the glacier, at all points not far from its surface, and near to the bottom of an ice-fall (art. 56). It appears to me inconceivable that any physical or mechanical effect (such as the lamination in question) really and primarily due to the difference of motion of contiguous particles should not manifest itself, if seen at all, in those directions in which the actual difference of motion is greatest. Assuming the truth of this conclusion, the structural surfaces must necessarily be such as above represented (art. 56). The investigation of the forms and positions of these surfaces depends only on the recognized motion of the glacier, and geometrical reasoning in which there can be no ambiguity. Now superficial curves of structure at the bottom of an ice-fall, as thus theoretically determined, are extremely elongated loops (art. 56); whereas it is one of the best-established of glacial phenomena that the actual superficial lines of structure at the foot of an ice-fall are nearly straight and perpendicular to the axis of the glacier. I maintain, therefore, that it is altogether impossible that the differential theory can be true, unless the expression "differential motion" has a very different meaning from any which I have been able to attribute to it*. On the contrary, the pressure theory, so far as it asserts the perpendicularity of the structural surfaces to the lines of maximum internal pressure, appears to be in accordance with all the observations which have yet been made on the subject.

SECTION VI.—*On the Intensity of the Forces employed in dislocating and crushing the masss of a Glacier.*

60. We have seen that, according to the Pressure Theory, when a glacier is brought into a constraint which it can no longer resist, its structure, or the continuity of its mass, must be destroyed in one of the three ways already specified (art. 28). It would seem probable that this is frequently effected by the crushing of the mass, the structure being immediately restored by regelation. But here arises the question, Whence do the internal pressures and tensions derive sufficient intensity to produce these crushing effects? The enormous weight of the mass of a glacier at once presents itself as the cause of the required intensity. But suppose, after the glacier has been broken and crushed, it should be instantaneously restored to perfect continuity of crystalline structure under the pressure to which, in any part, it may at the moment be subjected; why should the pressure, apparently the same as that under which it had, the instant before, regained its crystalline structure, again destroy the continuity of that structure? The highest mountain does not *crush* the strata which form its base. Again, the superincumbent weight can only produce any great effect at depths sufficiently great, whereas the effects of the internal pressures and tensions are exhibited in the most superficial

* With respect to Principal FORBES's conclusion, that a nearly horizontal force at the bottom of an ice-fall will produce a differential motion in nearly a vertical direction, I can only say that it appears to me so obviously opposed to the simplest conception of the action of such a force, as to be entirely inadmissible. It is unquestionably opposed to all accurate mechanical investigation on the subject (art. 53).

portions of the glacier, where the superincumbent weight can have no very sensible influence. These questions may be partly answered by a careful consideration of the theoretical explanations which have been given in the preceding pages; but for their complete answer they require, I conceive, the recognition of a kind of motion which has been till recently so entirely neglected. I allude to the motion by which a glacier *slides* over the bed on which it reposes (Sect. II. art. 7). I shall shortly proceed to explain the influence of this motion on the intensities of the internal pressures and tensions.

Principal FORBES was struck with this difficulty respecting the adequacy of the internal forces to produce the crushing effects ascribed to them, and the consequent mobility of the component particles *inter se*, though, regarding ice as viscous, the difficulty may naturally be supposed to have appeared less to him than if he had regarded it as a solid substance. He puts the difficulty in the following form*:—"Were a glacier composed of a solid crystalline cake, fitted or moulded to the mountain bed which it occupies like a lake tranquilly frozen, it would seem impossible to admit such a flexibility or yielding of parts as should permit any comparison to a fluid or semifluid body transmitting pressure horizontally, and whose parts might change their mutual position, so that one part should be pushed out whilst another remained behind." The difficulty as thus stated is equivalent to that above mentioned. Principal FORBES meets it as follows:— "But we know in point of fact that a glacier is a body very differently constituted. It is clearly proved by the experiments of AGASSIZ and others, that a glacier is not a mass of ice, but of ice and water, the latter percolating freely through the crevices of the former to all depths of the glacier; and as it is a matter of ocular demonstration that these crevices, though very minute, communicate freely with one another to great distances, the water with which they are filled communicates force also to great distances, and exercises a tremendous hydrostatic pressure to move onwards, in the direction in which gravity urges it, the vast porous crackling mass of seemingly rigid ice in which it is, as it were, bound up.

"Now the water in the crevices does not constitute the glacier, but only the principal vehicle of the force which acts on it; and the slow irresistible energy with which the icy mass moves onwards from hour to hour with a continuous march, bespeaks of itself a fluid pressure. But if the ice were not in some degree ductile or plastic, this pressure could never produce any, the least, forward motion of the mass. The pressures on the capillaries of the glacier can only tend to separate one particle from another, and thus produce tensions and compressions, *within the body of the glacier itself*, which yields, owing to its slightly ductile nature, in the direction of least resistance, retaining its continuity, or recovering it by reattachment after its parts have suffered a bruise, according to the violence of the action to which it has been exposed." The author again remarks (p. 167), "If it were not for the capillarity of the ducts, it is plain that no effective hydrostatic pressure could be developed at all; the flow being equal to the supply, no part of the *vis viva* would be expended in producing internal pressures."

* Occasional Papers, p. 165. See also a memoir in the Transactions of the Royal Society for 1846, Part III.

61. The author of the preceding extracts would appear, I think, to have somewhat mistaken the nature of the problem to be solved. He asserts that if a glacier were traversed by a great number of small tubes or ducts filled with water, an *enormous hydrostatic pressure* would be the consequence. Now it is absolutely necessary for me to examine strictly the correctness of this conclusion; for, if it be true, it would be useless to seek for any cause which should give efficiency to the internal pressures, besides the hydrostatic pressure to which it is here attributed. I shall show, however, that this conclusion is not correct, and that the efficiency of the dislocating forces is derived, as above intimated, from the sliding movement of the glacier. The existence of the above assumed capillary ducts, as pervading the more compact parts of a glacial mass, appears to be rendered doubtful by the experiments of Professor HUXLEY*, though the observations of M. AGASSIZ undoubtedly prove their existence in certain superficial portions of the glacier of the Aar. But waiving any doubt of this kind, let us suppose the interior of a glacier completely pervaded by these tubes or ducts, extremely small, but sufficient to allow fluid pressure to be communicated freely through them, as Principal FORBES has assumed. When the supply at the upper surface of the mass is regular, the motion of the fluid will become *steady*; and such I shall suppose it to be in the typical case I propose to analyse. We may first confine our attention to a single tube. Suppose the area (ω) of its transverse section at any point P to be variable, but always very small, and δs to denote the length of a small element of the tube; then will $\omega\delta s$ be the volume of the element, and, if the density of the water be denoted by unity, it will also represent the mass of the water in the element δs of the tube. Let the velocity at P, any point in the tube, be v, which I shall assume to be the same for each particle in the section (ω). The motion will be retarded by the *friction* of the sides of the tube; let $f\omega\delta s$ be the retarding force on the element at P. Also let p be the fluid pressure at P; and let the coordinates x, y, z of that point be taken as heretofore; then will z be very nearly vertical, and we shall have, resolving gravity in the direction of the tube,

$$\omega\delta p = g\,.\,\omega\delta s\,.\,\frac{dz}{ds} - f\,.\,\omega\delta s - \frac{d^2s}{dt^2}\,.\,\omega\delta s,$$

or

$$\delta p = g\delta z - f\delta s - \frac{d^2s}{dt^2}\delta s;$$

or, since the motion, by hypothesis, is steady,

$$\frac{d^2s}{dt^2} = v\frac{dv}{ds},$$

and therefore we have

$$p + C = gz - \int f ds - \tfrac{1}{2}v^2.$$

Let p_1, z_1, s_1 and v_1 be the values of p, z, s, and v at the point where the tube meets the surface of the mass. Then

$$p = p_1 + g(z - z_1) - \int_{s_1}^{s} f ds - \tfrac{1}{2}(v^2 - v_1^2), \quad \ldots\ldots\ldots \quad (1.)$$

where p_1 is the atmospheric pressure.

* See Dr. TYNDALL's 'Glaciers of the Alps.'

Also, since the same quantity of the fluid must pass through each transverse section of the tube in the same time, we must have

$$v\omega = v_1\omega_1,$$

and therefore

$$\tfrac{1}{2}(v^2 - v_1^2) = \frac{v^2}{2}\left(1 - \frac{\omega^2}{\omega_1^2}\right).$$

Let us interpret the above equation in different cases. We may first suppose the absence of all friction as a retarding force. We shall then have

$$p = p_1 + g(z - z_1) - \frac{v^2}{2}\left(1 - \frac{\omega^2}{\omega_1^2}\right);$$

and if the section (ω) of the tube do not contract so rapidly as to impede the motion of the water, the velocity acquired at the depth z will equal that of a body projected with velocity v_1, and falling freely by the action of gravity through the space $z - z_1$. We shall therefore have

$$\tfrac{1}{2}(v^2 - v_1^2) = g(z - z_1),$$

and therefore $p = p_1$ the atmospheric pressure, as if the tube were occupied by air. The least value of ω^2 for any value of z in this case will be given by

$$\omega^2 = \frac{v_1^2}{v^2}\omega_1^2$$

$$= \frac{v_1^2}{2g(z - z_1)} \cdot \omega_1^2.$$

If ω satisfy this condition for all values of z, the column of fluid will just fill the tube without its motion being impeded, while the pressure on the tube, as just shown, will be the atmospheric pressure p_1. But suppose ω to decrease, below a given section (Q), more rapidly as a function of z than is implied in the above equation. The motion of the fluid above the section Q would manifestly be retarded, and the retardation could only be due to the upward fluid pressure at Q being greater than the atmospheric pressure p_1 at the top of the tube. In like manner the increased downward fluid pressure at Q would accelerate the motion of the fluid immediately below that section. Any number of similar variations of ω, with consequent variations of velocity and fluid pressure, might take place, subject to the above condition that the tube shall always be full. The fluid pressure at any point cannot be less, but may be greater than p_1.

In this reasoning the tube is supposed to be independent of any other; but now suppose it to be confluent with a second and similar tube. At the point of junction we must have the condition that the pressures in the two tubes must be equal. This condition will be always satisfied, at whatever points in the surface of the mass the tubes may originate, provided each tube be such that the fluid pressure in it (as in the first case above explained) shall be equal at every point to the atmospheric pressure p_1. In such case water might pass from the upper surface through the mass, along any number of smooth tubes like those above described and communicating freely with each other.

The pressure in any tube would cause no reflux of the water, or other disturbance in a confluent one, because the pressures in them would be equal; but it must be carefully observed that there would, in this case, be no internal fluid pressure greater than that of the atmosphere, and therefore none capable of producing any effect in expanding the general mass, and urging it onwards. But if, on the contrary (as in the second case above explained), the tube be such that the pressure in each tube considered separately should be greater than p_1, the condition of an equality of pressure at the point of confluence of any two tubes would not be generally satisfied, and the whole motion would be interfered with. If the pressure p in any number of tubes became great, it would manifestly produce a reflux in any confluent tube in which the pressure was smaller, and a consequent outpouring of water from every pore and crevice on the surface of the lower regions of a glacier. Such, in fact, must necessarily be the case wherever there is an internal fluid pressure much exceeding that of the atmosphere, and where all parts of the mass are permeated by ducts of any kind through which water can flow out of the mass with little impediment.

Hence it appears that if the internal ducts were such as not to impede the currents passing through them by means of friction, or any kind of capillarity, there might indeed be large internal pressures, but they would necessarily be accompanied by overflowings from the superficial pores and crevices in many parts of a glacier. But such a phenomenon would be entirely opposed to all observation; and it is this which constitutes the proof that there can be no great fluid pressure in the interior of a glacier due to water contained within it in tubes which exert no sensible retarding effect by friction on the water running through them. This conclusion agrees with that expressed by Principal FORBES in the last of the preceding extracts; but the reason assigned is very different. The absence of great fluid pressure within the glacier in such case is not due to the absence of capillarity, but to the particular condition of its ducts and crevices being open in every part of the bounding surface of the mass. There can be no expenditure of *vis viva* in producing pressure on a *smooth* surface, as is well known, though such pressure must necessarily be produced in the case before us, as in all cases of constrained motion when the constraint is produced by the action of such surfaces. The fact of no *vis viva* being lost does not imply that no internal pressure would be produced.

62. But let us now turn to the far more important case in which the passage of the water through the small ducts of any mass, as that of a glacier, is impeded by the friction of the sides of the ducts. It follows, as shown in the preceding explanation, that, since in actual cases there is no overflowing of water from the pores and crevices, here assumed to pervade one part of the mass as well as another, the fluid pressure in all confluent ducts at the point of confluence must be the same, and, therefore, independent of the height above that point at which each duct may meet the external surface of the glacier. This pressure, when the surface is exposed to the atmosphere, can be only the atmospheric pressure. Hence it follows in this case, as in the previous one, that there can be no internal fluid pressure greater than that of the surrounding

atmosphere, and such, therefore, as can exert any influence in expanding the mass, and promoting its onward motion. It will act on the interior of the mass in directions *normal* to the tubes. But there is another force also, friction, acting on the mass of the glacier in this case, in directions *tangential* to the tubes, the magnitude of which requires to be considered. It retards the motion of the fluid, acting equally on the fluid and on the sides of the tubes containing it in opposite directions. Now in the case before us, $p=p_1$, the atmospheric pressure, and, therefore, by equation (1.), art. 61,

$$\int_{s_1}^{s} f ds = g(z-z_1) - \tfrac{1}{2}(v^2 - v_1^2);$$

and differentiating,

$$f = g\frac{dz}{ds} - \tfrac{1}{2}\cdot\frac{dv^2}{ds},$$

and

$$\int_0^{s_1} f\omega ds = \int_0^{s_1} g\omega ds\,.\frac{dz}{ds} - \tfrac{1}{2}\int_0^{s_1}\frac{dv^2}{ds}\,.\,\omega ds, \quad \ldots\ldots \quad (2.)$$

where $\int_0^{s_1} f\omega ds$ is the whole retarding force of friction in any one of the tubes whose length $=s_1$ (art. 61), and is therefore the amount of friction produced by the water on the sides of the whole tube.

To find the value of the last term in equation (2.), we have (the duct being always full)

$$v\omega = v_1\omega_1, \quad \ldots\ldots \quad (3.)$$

where ω_1 is the value of ω when $s=0$ and $v=v_1$. Therefore

$$v^2 = \frac{v_1^2\omega_1^2}{\omega^2},$$

$$\frac{dv^2}{ds} = -\frac{2v_1^2\omega_1^2}{\omega^3}\cdot\frac{d\omega}{ds},$$

$$\frac{dv^2}{ds}\omega\delta s = -\frac{2v_1^2\omega_1^2}{\omega^2}\cdot\delta\omega,$$

and

$$\int_0^{s_1}\frac{dv^2}{ds}\omega\delta s = 2v_1^2\omega_1^2\left(\frac{1}{\omega_2} - \frac{1}{\omega_1}\right).$$

Now we may venture to assert that when water descends through exceedingly minute ducts in the manner here supposed, the velocity with which it will permeate the lower portions of the mass will be much the same as that with which it will pass through the upper portion. Assuming this, we shall have by equation (3.) $\omega_2=\omega_1$, and the value of the above definite integral will $=0$. Hence, by equation (2.), we have

$$\int_0^{s_1} f\omega ds = \int_0^{s_1} g\,.\,\omega ds\,.\frac{dz}{ds}:$$

$g\omega ds$ is the weight of an element of the water in the tube, and $g\omega ds.\frac{dz}{ds}$ is a force less than that weight. Consequently the definite integral on the right-hand side of the last

equation expresses a force *less* than the weight of the whole fluid contained in the tube; and if we call this last weight W_1, that equation shows that the entire effect of the friction, estimated in the tangential direction in which it acts at each point of the tube, is less than W_1.

Again, let δX_1 be the part of the friction on any element of the tube, resolved parallel to the axis of x; then

$$\delta X_1 = f\omega ds . \frac{dx}{ds},$$

$$X_1 = \int_0^{s_1} f\omega ds . \frac{dx}{ds} = \int_0^{s_1} g\omega ds . \frac{dz}{ds} . \frac{dx}{ds};$$

similarly, we have

$$Y_1 = \int_0^{s_1} f\omega ds . \frac{dy}{ds} = \int_0^{s_1} g\omega ds . \frac{dz}{ds} . \frac{dy}{ds},$$

$$Z_1 = \int_0^{s_1} f\omega ds . \frac{dz}{ds} = \int_0^{s_1} g\omega ds . \left(\frac{dz}{ds}\right)^2.$$

Since $\frac{dx}{ds}$, $\frac{dy}{ds}$, and $\frac{dz}{ds}$ are each less than unity, it follows that X_1, Y_1, and Z_1 are each less than W_1; and if, taking all the tubes,

$$X = X_1 + X_2 + \&c., \quad Y = Y_1 + Y_2 + \&c., \quad Z = Z_1 + Z_2 + \&c., \quad W = W_1 + W_2 + \&c.,$$

we shall have X, Y, and Z each less than W, the weight of water in the whole mass of the glacier.

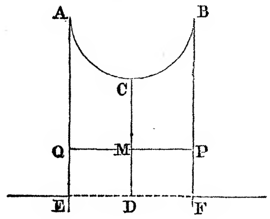
The internal pressure Z will tend to elevate the upper surface of the glacier; the pressure Y will be equilibrated by the pressures on the opposite flanking walls of the glacial valley. To interpret the meaning of X, we may suppose the vertical section of the glacier near its origin to be immoveable; then will the pressure X tend to urge onwards the whole mass in the direction of the containing valley, where there is no barrier to oppose its action.

To form a conception of the magnitudes of the pressures X, Y, Z, we may remark that the water in our supposed tubes must almost necessarily descend through the mass in approximately vertical directions. Admitting this supposition, $\frac{dz}{ds}$ will nearly equal unity, and Z_1 will nearly equal $\int_0^{s_1} g\omega ds$, or nearly the weight of the fluid contained in the single tube. Consequently the whole vertical resolved part of the friction $=Z=$ approximately the whole weight of the fluid contained in the glacier. Y will be equal only to a small portion of W, because $\frac{dy}{ds}$, in the case we have taken, will be very small; and X will be also very small, because $\frac{dx}{ds}$ will be so. And thus it follows that the whole internal pressure, arising from the friction on the sides of the small ducts through which the water descends, and tending to urge the glacier forwards, cannot exceed a small fraction of the weight of the water descending through its pores, and this latter weight again must doubtless be an exceedingly small fraction of the weight of the glacier.

63. It appears, then, from these explanations, that there will be at every point (P) of every duct, two internal forces, the fluid pressure p acting normally, and the friction acting tangentially at that point on the sides of the tube. But assuming these small ducts to pervade the superficial as well as the inner portions of the mass, the fluid pressure p can never much exceed the atmospheric pressure, which, acting externally, will very nearly counterbalance it; and this will be true whether the motion of the water in the ducts be impeded by friction or not. Also the whole resolved part of friction acting on the ducts, in the direction of the general motion of the mass, must be very small compared with the weight of the whole percolating fluid. Neither of these forces, therefore, can have any sensible influence in augmenting the onward motion of a glacier, compared with other forces which, as I shall shortly explain, tend to produce that effect.

64. In the preceding explanations I have spoken of the retardation of the motion of a fluid in small ducts, as due to friction rather than capillary action. Capillarity, when it exhibits itself in the form under which it is more usually contemplated, acts in a manner totally different to that in which friction acts. We know that a column of water, or of other fluids, of a certain length may be supported in a vertical tube of sufficiently small bore, by the attractions between the particles of the fluid, and the attraction between the fluid and the tube. These attractions produce the capillarity of the tube. If, however, the tube be perfectly full, whether the fluid be at rest or in motion, its capillary action on the fluid will counteract itself, and will produce no effect sensible to observation. It is only when the tube is partially empty that a part of the capillary action on the contained fluid is uncompensated, and exhibits itself in the column of fluid which it is capable of supporting. These partially empty tubes cannot properly belong to a system of tubes, like those of a glacier, which serve as *ducts* through which the water is constantly running, and which must therefore be constantly full; at all events, the number of tubes exhibiting the effect of capillarity as above described must probably be extremely rare in a system of ducts like those of a glacier. Supposing such tubes, however, to exist, we proceed to explain their effect in producing internal pressure.

Fig. 18.



Let A E F B be a section of the capillary tube made by a plane through its axis, the tube being considered cylindrical and vertical. Let E D F be the level of the surface of the external fluid, which we may suppose to be water, and into which the lower end of the tube is placed. Also let A C B be a section, by the above plane, of the surface of the water maintained in the tube by capillary action. Our object is to ascertain whether this water will produce any pressure on the sides of the tube tending to thrust them outwards, and thus produce an expansion of a solid mass in which any number of such capillary tubes might exist.

For this purpose we must consider how the column of water in the tube is supported.

Now, manifestly, the atmospheric pressure, acting equally in all directions, can contribute nothing to this effect; nor can there be an upward pressure on the base E F of the raised column of water; for the pressure at any point of that base can only be equal to that at any point in the surface of the external fluid, which, neglecting the pressure of the atmosphere (as we may do from what has just been stated), will be zero. Again, the resultant attraction of the tube on any single particle of the fluid must be in a direction perpendicular to the surface of the tube, and must therefore be horizontal. Consequently it can produce *directly* no force on the particle or on the whole mass of the fluid column in a vertical direction. The direct effect of gravity, on the contrary, is to drag the whole column of water downwards, and the leading point in the problem of capillary action is to explain how this tendency of gravity is counteracted.

In a question like this, which is merely subsidiary to the general problem treated on in this paper, it must suffice that I quote those results which are familiar to every one acquainted with the ordinary investigations respecting capillary action. Now it is shown by such investigations that the fluid column A E F B is supported (so far as regards any vertical action upon it) in the same manner *as if* it were suspended freely from an infinitely thin and perfectly flexible membrane accurately coinciding with the actual surface A C B, the particles of the fluid being supposed to adhere to the membrane and to each other by virtue of the cohesion or attraction between them, without which, in fact, the ordinary phenomena of capillarity could not exist.

If the column of water in the tube were entirely supported by an upward pressure on its base E F, then would gravity produce a *pressure* on any horizontal section Q M P, the whole amount of which would be the weight of the portion of the column above it, and there would be a corresponding *pressure* on the inner surface of the tube at P and Q, tending to push it *outwards*; but if the column were supported, as above supposed, at its upper surface, gravity would produce a *tension* at the horizontal plane Q M P, the whole amount of which would be equal to the weight of the fluid between Q P and the base E F. Thus the whole column would be in a state of *longitudinal tension*, and therefore also, by the fundamental property of fluids, in a state of *horizontal tension*—*i. e.* the action between the fluid and the inner surface of the solid tube, instead of being a pressure, must be a *tension*, supported by the mutual attraction between the particles of the solid and those of the fluid, supposing the contact between them not to be broken. This must be the direct effect of gravity in the actual case; and so far, therefore, its tendency is to contract and not to enlarge the diameter of the tube.

There are other causes, however, by which action may be produced between the tube and the contained fluid. First, a *pressure* will necessarily arise from the mutual attraction between the tube and the fluid; but here we have the action of the fluid on the tube, and the reaction of the tube on the fluid, the one tending to pull the surface of the tube inwards, and the other to push it outwards. The action and reaction thus counteract each other, and the pressure thus produced can manifestly have no tendency either to expand or contract the tube.

Again, in any actual case of a fluid sustained in a capillary tube, the inner surface of the tube will not generally be a surface of free equilibrium like the upper surface of the fluid itself; and therefore there must necessarily be some action between the tube and fluid, should they remain in contact, even if there were no attraction, like that just considered, between them. Now it can be shown, according to the fundamental principles on which the theory of capillarity is founded, that in all cases resembling the one we have been considering, the action in question must be equivalent to a *tension* on the inner surface of the tube. Hence this action, as well as that due to gravity, produces forces on the tube which tend to *contract* and not to *expand* it, while the mutual attractions of the tube and fluid produce directly neither the one tendency nor the other. Consequently the general tendency of capillary action in the interior of a glacier will be to contract and not to expand its dimensions. The contrary notion has probably arisen from the idea that gravity would produce a pressure on the interior surface of a capillary tube like that which it produces in a tube of larger dimensions. The fluid which may exist in these capillary tubes will, of course, add its own weight to that of the glacier, an addition probably too insignificant to produce the slightest sensible influence.

65. There is something so plausible at first sight in the idea of a great internal fluid pressure within a glacier, due to the water percolating through it, or suspended within it by capillarity, while additional weight has been given to this notion by the advocacy of the author of the Viscous Theory, that I have thought it essential to enter into more details on the subject than I should otherwise have deemed necessary, in order to prove the fallacy of the conclusions which have been deduced from erroneous views respecting it.

I now proceed to explain how the intensity of the internal forces is increased by the sliding movement of the glacier.

66. *Effect of the Sliding of a Glacier on the Internal Pressures and Tensions.*—It has frequently been objected to the sliding of a glacier, that the inequalities along the sides of the containing valley would effectually prevent such motion. This objection, however, will be entirely obviated, if we eliminate the uncertain and irregular opposing forces arising from local lateral obstacles, by substituting for the sides of the glacial valley two imaginary vertical planes near and approximately parallel to them, so that we shall thus have the tangential action of ice along these planes as the retarding force on the general mass of the glacier, instead of the action of the walls of the valley. The greatest magnitude of this retarding force must be equal to the greatest tangential cohesive power of the ice. This is, in fact, the greatest force which the walls, however irregular, could possibly exercise in opposing the sliding past them of the general mass of the glacier. To simplify our problem as much as possible, I shall also first suppose the glacier of uniform width and thickness, and the inclination of its bed to be likewise uniform. We may then consider A and B each $=0$. I shall also suppose $C=0$, and

$E=0$, as well as $D=0$. We shall thus have (art. 29 (1.)),

$$p_1=F, \quad p_2=-F,$$
$$\tan 2\alpha=\infty.$$

F will vanish along the axis of the glacier, as we have seen, and will have its greatest value along the imaginary vertical planes by which we here consider the moving mass as effectively bounded along its lateral margins. Let its value along these planes be F_1. If we suppose, as we may in this approximation, that every particle in the same vertical line moves with the same velocity, F_1 will be the same for every point of the bounding lateral planes. Moreover, let the tangential action exercised by the bed of the glacier on its lower surface be denoted by f_1. Let us now conceive the mass to be placed in a position of no constraint. It will immediately assume a position of constraint by virtue of its small extensibility, and, provided the internal forces p_1, p_2, and F are insufficient to dislocate the mass along the lateral vertical planes (where their magnitudes will be the greatest), the glacier will be held at rest in its state of constraint by the external forces acting upon it. Now F_1 and f_1 being referred to a unit of surface, if a, b, c be the length, breadth, and depth of the glacier, we shall have

$$\text{The tangential force on each flank } = F_1 ac,$$
$$\text{The tangential force on the bottom } = f_1 ab,$$

and these must be in equilibrium with the weight of the mass on the plane whose inclination is ι. Hence if ω be the weight of a unit of volume of glacial ice, we shall have

$$2F_1 ac+f_1 ab=\omega abc \sin \iota,$$

and

$$F_1=\tfrac{1}{2}\left\{\omega b \sin \iota - f_1 \frac{b}{c}\right\}.$$

Suppose $\bar{F}$ to denote the tangential cohesive power of the mass. Then, if

$$F_1=\bar{F},$$

the glacier will be just on the point of dislocation; and if F_1 exceed $\bar{F}$ in the smallest degree, the mass will be dislocated along the lateral planes and move onwards.

We have shown (art. 8) that the effect of f_1 *to hold the mass of the glacier at rest* is extremely small; consequently it may be neglected in the above equation, and we shall then have

$$F_1=\tfrac{1}{2}\,\omega b \sin \iota.$$

If the lower surface of the glacier were firmly frozen to its bed, we might have $f_1=F_1$, and therefore

$$F_1=\frac{\omega b \sin c}{2+\frac{b}{c}}:$$

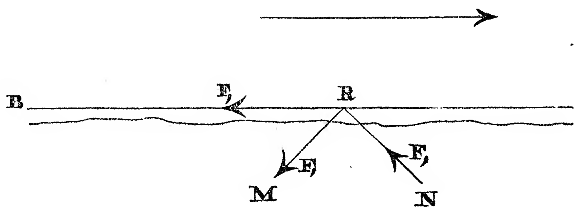
c is properly the depth of the glacier along its flanks, so that $\frac{b}{c}$ may be large.

Hence we see that F_1 may become much greater when f_1 is very small, than if it were

nearly $=F_1$. Consequently F_1 *will be much more likely to attain the magnitude* $\overline{F}$, *sufficient to dislocate the mass, if the glacier slide over its bed as described in art.* 8, *than if it were attached to its bed so as not to slide at all, and its whole motion should be that due to the molecular mobility of its particles alone.*

The above equation expresses the condition of the mass being on the point of dislocation in terms involving the *tangential* cohesion; it will be easy to express it in terms of the *normal* cohesion. Generally, instead of the forces A, B, F acting on any element, we may substitute the forces p_1, p_2 (art. 29), acting in their proper directions as determined by the values of α. Let fig. 19 represent one side of the glacier with the lateral plane B R; then, since at any point (R) in that plane $p_1=F_1$, $p_2=-F_1$, and $\tan 2\alpha=\infty$, we may substitute for F_1 acting tangentially along R B, two other forces, viz. a tension $=F_1$ along R M, and a pressure $=F_1$ along N R, both directions making angles of 45° with B R. Consequently an equal tension and pressure similarly applied at each point of the lateral vertical planes, would by hypothesis balance the tendency of the mass to descend in the direction B R. Now F_1 acting at an angle θ on B R will produce a force $=F_1 \cos\theta$ on a unit of surface of B R. Therefore the tension F_1 acting along R M will produce a force on the same unit, of which the resolved part along R B will

Fig. 19.



$$=F_1 \cos 45° . \cos 45°,$$

$$=\frac{F_1}{2}.$$

Similarly, the pressure F_1 along N R will produce a force the resolved part of which along R B will also $=\frac{F_1}{2}$. These together $=F_1$, and the whole supporting force on the two lateral vertical planes will $=2F_1 ac$, F_1 being now the measure of a pressure or a tension acting *normally* on a unit of surface. Therefore

$$2F_1 ac=\omega abc \sin\iota,$$

and

$$F_1=\omega\frac{b}{2}\sin\iota.$$

Consequently a glacier like that we have been considering, will necessarily be dislocated by its own weight resolved along the plane of its bed, if its normal cohesive power be less than the weight of a column of glacial ice whose transverse section is unity, and whose length = semiwidth of the glacier multiplied by the sine of the inclination of its bed to the horizon. If the glacier were a mile wide and its inclination about 5°, it would be necessary, in order that it should not be dislocated, that its cohesive power should be such that a vertical cylindrical column of glacial ice about 200 feet long should be capable of supporting itself when suspended by its upper extremity. This would be the measure of the greatest tension $p_1(=F_1)$ at any point along either of the

lateral vertical planes, which could be produced under our assumed conditions. If the cohesion were less than this, the mass would necessarily be ruptured in some way or other in its marginal regions. The above, however, is only the lowest limit to which the force F_1 would attain. For let a represent the length of a given portion only of a glacier, such that at the lower transverse vertical section of that portion, the longitudinal *tension* shall be greater than at the higher bounding transverse section, instead of being equal to it as above supposed. It is manifest that F_1 will be increased along the lateral vertical planes of that portion of the glacier. Such will also be the case if the same portion of the glacier be acted on by a longitudinal *pressure* on its higher transverse bounding section, greater than that on its lower one. There would generally, in such cases, be a much greater effort to overcome the cohesion $(\overline{F})$ of the mass along the lateral vertical planes than in the case previously considered. If the valley of the glacier contract somewhat rapidly, the longitudinal pressure may be enormously increased on a given portion of the glacier by the action of the mass behind it, and this increased action will manifestly depend very much on the facility with which the mass behind slides over its bed.

It may be well to take another numerical example in which the conditions are similar to those of the example given above. Let the glacier be something less than half a mile broad, and its inclination about 3°. Then it appears, from the above expression for F_1, that dislocation would not take place, provided the cohesion of the mass were not less than the weight of a column of ice of about 60 feet long, instead of 200 as in the former example. This supposes the mass to slide freely over its bed, and the longitudinal pressure or tension to be the same behind as before; but if we suppose the mass to adhere to its bed, that adherence will help the tangential action along the lateral vertical planes to support the mass. Consequently the force F_1 called into play might be much less than the weight of a column 60 feet long, and might not be sufficient to overcome the cohesion $\overline{F}$, in which case the motion of the mass would be arrested. This conclusion would be strictly applicable only to our *typical* glacier, but is sufficient to explain how, in the actual case of a glacier, the facility of its sliding may increase its power to overcome the obstacles to its motion by the fracturing of its mass.

I have chosen the simplest cases for the purpose of more easily explaining the manner in which the sliding of a glacier increases the dislocating power of the forces acting upon it. In the more complicated cases, in which C and E are taken into account at considerable depths, we shall obtain greater values of the tearing and crushing forces p_1 and p_2. It is the latter which will be principally increased at greater depths. Still it must not be supposed that the mere increase of weight can explain the apparent difficulty already suggested (art. 60), viz., Why, when the mass has been crushed and then immediately regealed under the existing conditions at any proposed point, it should again be crushed at the same point. This second crushing does not, in fact, take place under the same pressure as that under which the regelation took place. This latter process occurs when the pressure depending on the angular distortion of the element, or on the force F, has

been destroyed by the previous crushing; and then the angular distortion, the force F, and the increase of the maximum pressure are reproduced, and the mass is crushed again. We thus understand how the alternate processes of crushing and regelation must be repeated when the mass is in motion, though the weight alone of the mass, however great, might not, and probably would not, if unaided by the distortion arising from the motion, be sufficient to crush any even the lowest portions of the glacier, any more than the superincumbent weight of a mountain crushes the lower portion of the strata which compose it. We have seen how this power of distortion is increased by the sliding of the glacier.

It is thus that I conceive the process of regelation in glaciers to be intimately united with their uniform sliding movement. Both these properties are proved to belong to ice at the same particular temperature (that of freezing), by clear and conclusive experiments, and form, as I have endeavoured to show, the foundation of a theory which explains all the principal observed phenomena in the motion of glaciers, without assigning to ice any property, like viscosity accurately defined, which it cannot be experimentally proved to possess. In the introduction to this paper I have shown that the property of regelation is totally distinct from that of plasticity or viscosity, if by either of those terms it is intended to designate a determinate property of bodies distinct from that of solidity. All the experimental elucidations which have been given of the Viscous Theory have been drawn from bodies, such as moist plaster of Paris, tar, treacle, unconsolidated lava, &c., all of which may with strict propriety be termed plastic or viscous; but I maintain that ice possesses no property which at all assimilates it to those substances. I have shown that the *apparent* plasticity of ice, as manifested in the compressions of glaciers under great pressure in the course of their motion, is no determinate property of ice at all, but a consequence of its fracture and regelation. Still, differing as I do from the author of the Viscous Theory, I consider the scientific world largely indebted to him for his unwearied researches respecting glacial phenomena, for the large amount of general and detailed knowledge which he has communicated to us, and the interest with which he has helped to invest the subject. Moreover, though he may not have been the first to observe that fundamental character in the motion of a glacier which consists in the more rapid motion of its axial portions, he was the first glacialist whose mind was thoroughly imbued with the necessity of recognizing the influence of molecular mobility in the mass of a glacier on its general onward motion. In like manner, though not absolutely the first to observe that interesting and characteristic structure of glacial ice, the veined structure, he was doubtless the first to recognize the existence of law in that structure, and its consequent importance as indicating the operation of causes physical or mechanical, or both, with which we were entirely unacquainted. In all this, Principal FORBES has contributed largely to the sound progress of glacial science, and his labours must always be highly appreciated. Beyond this boundary, he stepped into the region of speculative theory; and it appears to me, as may be seen from much I have said in this paper, that he did so without any distinct definition of the

fundamental physical principle on which his Viscous Theory rested, and without the experimental verification which it demanded, and, moreover, without the guidance of those more refined and exact mechanical researches without which I am sure we can see our way but dimly through the more complicated of those problems which glacial theory presents to us. Hence it is that I have been led to dissent from most of his speculative views. That dissent has been unreservedly expressed in this paper, and therefore it is that, in concluding it, I would bear testimony to what I consider the great and legitimate claims which the author of the Viscous Theory (however we may differ from the theory itself) has established to be regarded as one of the leading promoters of glacial science.

After the greater part of this paper was written, a very interesting memoir by the Master of the Mint, "On Liquid Diffusion applied to Analysis," was brought under my notice. He there speaks of the "colloidal condition of matter" (of which gelatine seems to afford the type) as opposed to the "crystalloidal condition." He remarks (Philosophical Transactions, Vol. 151, Part I. 1861), "Ice itself presents colloidal properties at or near its melting-point, paradoxical though the statement may appear." Again, "Ice, although exhibiting none of the viscous softness of pitch, has the elasticity and tendency to rend seen in colloids." "It further appears to be of the class of adhesive colloids. The redintegration (regelation of FARADAY) of masses of melting ice when placed in contact, has much of a colloid character. A colloidal view of the plasticity of ice demonstrated in the glacier movements will readily develope itself."

These passages were written without any direct reference to glacial questions; but it occurred to me that a glacialist might possibly put constructions on them unfavourable to the views I have expressed respecting the *solidity* of ice, and the absence in that substance of any property which could, according to my own definition of the term, be called *plastic*. Consequently I wrote to the author requesting him to give me some further elucidation of his views on one or two points bearing on my own definitions and explanations. I proposed to him the following questions, to which he kindly gave me the subjoined clear and explicit replies.

(1) "Is the tendency to a colloidal character in ice, as opposed to a vitreous, crystalline brittle structure, sufficient to interfere materially with the *restitution* described*, by giving the ice a greater degree of plasticity?" Mr. GRAHAM'S answer was, "I believe not. A colloid, on the contrary, is often as nearly perfectly elastic as possible. Take the gluey material used to form the roller by which ink is applied in book-printing, as an illustration of the elasticity of a colloid, with the entire absence (apparently) of true plasticity or viscosity."

(2) "Will the colloidal state of ice at temperature = zero (C.) materially modify the *modus operandi* (assuming the ice to be solid), rendering it approximate to the process which would take place supposing the mass to be plastic?" Mr. GRAHAM replies,

* See art. 1.

"Quite the contrary. The only influence to be looked for from the colloidal state would be greater elasticity—that is, the matter would be likely to yield more before rending or breaking, supposing it to be colloidal, than if it were crystalline. Your mechanical conception of the *modus operandi* in the fracture of a solid (not plastic) applies equally well to a crystal or colloid. I go entirely with you up to that point."

Mr. GRAHAM further remarks, "Colloidality is invoked chiefly with the view of accounting for the ready reunion, the redintegration as I have called it, of fragments of ice brought into contact with each other. It is a very general (perhaps universal) character of colloids to adhere and reunite when two masses are pressed together. In fact all our adhesive substances, gum, glue, starch, &c., belong to the class of colloids. Even glass, which is a colloid of fusion, shows the adhesive character, two sheets of polished plate glass often adhering so thoroughly as to tear up each other's surface when forcibly separated. Another colloid adhesive like ice, is fused phosphoric acid—'glacial' phosphoric acid as it is called, in prescience, one might imagine, of this discussion! No such adhesive property is ever found in crystalline surfaces, so far as I am aware. It is *quà* colloid that ice appears to be adhesive. The discovery of FARADAY's, of the adhesive quality of ice, is the fundamental fact of the glacier discussion. The name 'regelation' applied to it may, however, be objected to, being quite speculative, and implying, as it appears to do, that two pieces of ice come to be *cemented together* by the freezing of a film of water between them, instead of simply adhering perfectly and uniting as two pieces of plate glass might do. The great fact, however, remains, and the name is but a trivial matter."

I also thought it right to inquire of Mr. GRAHAM the exact meaning he intended to attach to the word "demonstrated" in the last sentence of the above quotation from his memoir. A glacialist might possibly, I thought, if so disposed, interpret it as meaning "proved" or "fully established." In his answer Mr. GRAHAM says, "For 'demonstrated' I should have said 'indicated' as you suggest." He also remarks, "You will, I am certain, do good service in the glacier discussion by the precision you introduce into the terminology of the subject*. I accept without reserve your definitions of plasticity, &c. In the passage you quote from my paper, I had spoken of ice as plastic in a more general sense, as plasticity is understood, I apprehend, by Professor FORBES†. A solid was looked upon as plastic of which the form can be remodelled *anyhow*—by fracture and reunion, as well as in consequence of viscosity."

These views of the Master of the Mint are so explicitly and clearly expressed that they need no comments on my part, beyond the expression of the high gratification I derive from the coincidence of the opinions stated in his letter with those which have been put forth in this memoir.

* I had communicated my definitions to Mr. GRAHAM.

† This is what I termed *quasi-plasticity*, or *apparent plasticity*.